Determination of the temperature of thermally unprotected steel members under fire situations. considerations on the section factor

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Abstract

This work provides a derivation of an expression, usually found in fire steel structures Standards, like NBR 14323:1999 [1] and Eurocode 3 [2]. It allows calculating the temperature of thermally uninsulated steel members. Within these standards, the limits of the utilization of the expression are not clearly stated. The main contribution of this work is to clarify this subject. The adopted hypotheses are detailed in order to understand its use. With the aid of the Super Tempcalc computer software, the situation of a steel plate protected by a concrete slab over one face is thermally analyzed. Considerations are made on the determination of the section factor in other situations that, despite being usually found in civil construction, do not follow the hypotheses of the regarded expression.

Keywords: fire, fire safety, steel, temperature, thermal analysis

1 Introduction

1.1 Objective

The objective of this work is to present considerations on the limitations of the use of the well-known expression for calculation of the temperature of thermally unprotected steel members under fire situations. For this purpose, the derivation of the expression is detailed, as well as the adopted hypotheses. Comments are made on section factor geometric characteristic and its use in usually found situations in civil construction that are not according to the hypotheses that were adopted in the derivation of the expression.

1.2 General considerations

Thermal action is the action on the structure described by heat flux (\( \dot{Q} \)), by radiation and convection, caused by the temperature difference between the hot
gases and structure members. The temperature rise on structure members, due to thermal action, causes reducing of strength and modulus of elasticity and additional loads (indirect actions) wherever there are restraints to thermal deformations.

The safety conditions of structures within a building in a fire situation are verified when the temperature of structural members during the fire is less then the critical temperature of these members, where critical temperature is the temperature that causes the structure to collapse [3, 4].

Temperature at the structural member can be experimentally or analytically determined. In fact, there is neither a solely experimental method nor a purely theoretical one. The so-called experimental methods bear on simplified hypotheses (e.g. use a fictitious time-temperature standardized curve [5]) and depend on the fine gauging of the heating furnace, sometimes checked by comparison with theoretical results. The so-called analytical processes, by their turn, rely on experimental proofing of the employed parameters. In this article, an analytical method for the determination of temperature of thermally unprotected steel structure members will be studied.

2 Temperature determinations

The temperature in thermally unprotected structure steel members exposed to fire can be determined by means of the well known expression 1.

\[ \Delta \theta_a = \frac{F}{c \rho} \Delta t \]  

(1)

It is intended within this work to derive the expression 1 and detail the adopted hypotheses, having in mind that they are fundamental ones in order to admit the use limitations of expression 1.

Initially, one admits one-dimensional flux and thermal conductivity of steel high enough so that, by safety, one considers that the temperature spreads instantly along all the length of the member under study.

The temperature difference between the fire flames and the structural members generates a heat flux that, via radiation and convection, transfers itself to the structure causing temperature rise. The temperature rise in the structural member is determined considering the thermal balance between the heat emitted by the fire and that absorbed by the steel part (Fig. 1).

\[ \theta_g \rightarrow Q \rightarrow \theta_a \rightarrow Q_{abs} \rightarrow \theta_a \rightarrow Q \rightarrow \theta_g \]

Figure 1: Heat flux

Convection is the process in which the heat flows, involving moving of fluid mix, mainly between solids and fluids. The heat flux by convention is generated by the
difference of density between the gases within the flaming compartment; the hot gases are less dense and tend to occupy the superior atmosphere, while the cold ones, of higher density have the tendency to move to the lower compartment’s atmosphere. A contact between hot gases and the structure, from this motion, yield the heat transfer. The expression for the calculation of convective heat flux is due to I. Newton in 1701. Exp. 2 is already adapted to the regarded problem.

\[ \dot{Q}_{c} = h_{c} \cdot A_{a} \]  

(2)

where:

\[ h_{c} = \alpha_{c} \left( \theta_{g} - \theta_{a} \right) \]

Radiation is the process by which the heat flows in the form of wave propagation, from one body at high temperature to another one at lower temperature. The expression for the calculation of the heat flux generated by an ideal radiator (black body) was experimentally found by J. Stefan in 1879 and theoretically derived by L. Boltzmann in 1884 (exp. 3).

\[ \dot{Q}_{r} = \sigma \cdot A \cdot (\theta + 273)^{4} \]  

(3)

For the case of heat exchange between two real bodies (not ideal) one has exp. 4.

\[ \dot{Q}_{r} = \sigma \cdot A \cdot \varepsilon \cdot 1 - 2 \left[ (\theta_{1} + 273)^{4} - (\theta_{2} + 273)^{4} \right] \]  

(4)

Rewriting exp. 4 in an adequate manner to the referenced problem, one has exp. 5

\[ \dot{Q}_{r} = h_{r} \cdot A_{a} \]  

(5)

where \( h_{r} \) and \( \alpha_{r} \) are calculated by expressions 6 and 7.

\[ h_{r} = \alpha_{r} \left( \theta_{g} - \theta_{a} \right) \]  

(6)

\[ \alpha_{r} = \frac{\sigma \cdot \varepsilon \cdot \left[ (\theta_{g} + 273)^{4} - (\theta_{a} + 273)^{4} \right]}{(\theta_{g} - \theta_{a})} \]  

(7)

So, using expressions 2 and 5 results exp. 8

\[ \dot{Q} = \dot{Q}_{r} + \dot{Q}_{c} = \dot{h} \cdot A_{a} \]  

(8)

where:

\[ \dot{h} = \dot{h}_{r} + \dot{h}_{c} = \alpha \left( \theta_{g} - \theta_{a} \right) \]

According to NBR 8681:2004 [6], the combination factor associated to the accidental action is equal to 1.0. Hence, we have:

\[ \dot{Q}_{d} = 1.0 \cdot \dot{Q}_{k} \text{ and } h_{d} = 1.0 \cdot h_{k} \]
Within this article, the notation will be simplified and it will be used $\dot{Q}$ and $\dot{h}$ instead of $\dot{Q}_d$ and $\dot{h}_d$.

The heat absorbed by the steel member in the unity of time is determined by exp. 9.

$$\dot{Q}_{abs} = m c(\theta_{a,vol}) \dot{\theta}_{a,vol} = V \rho c(\theta_{a,vol}) \dot{\theta}_{a,vol}$$

(9)

Considering the thermal balance, we obtain exp. 11 by means of exp. 10.

$$\dot{Q} = \dot{Q}_{abs}$$

(10)

$$\dot{h} A_g = V \rho c(\theta_{a,vol}) \dot{\theta}_{a,vol}$$

(11)

Supposing that $\dot{\theta}_{a,vol} \approx \frac{\Delta \theta_{a,vol}}{\Delta t}$ and defining the section factor ($F$) as the relation between the fire exposed area ($A_a$) and the heated volume of the steel element ($V$), we get the exp. 12.

$$\theta_{a,vol}(t + \Delta t) - \theta_{a,vol}(t) = \frac{F}{c(\theta_{a,vol})\rho} \sigma e \left[ (\theta_g(t) + 273)^4 - (\theta_a(t) + 273)^4 \right] +$$

$$\Delta \theta_a(t) \Delta \theta_a(t)$$

(12)

One imposes, now, a sometimes forgotten important hypothesis, that is, the exp. 13.

$$\theta_{a,vol} = \theta_a$$

(13)

Finally, from expressions 10 to 13, one gets exp. 1.

Where:

$\Delta \theta_a = \theta_a(t + \Delta t) - \theta_a(t)$

The hypothesis represented by exp. 13 means that the average temperature in the volume must be equal to the average temperature of the surface exposed to fire. This happens to members formed by thin walls that do not have contact with other heavy members, like concrete or masonry, that could absorb heat. If this situation occurs, there can be differences between the temperatures in the volume and the surface. Some parts of the section could reach temperatures higher than the expected, leading to local problems that, by their turn, could lead to global ones.

Figure 2 presents the temperature curves of steel members with Section Factors 50, 75, 100, 125, 150, 175, 200, 250 and 300 m$^{-1}$, obtained by applying exp. 1 to the standard fire model [5]. It was supposed the specific heat of steel varying with the temperature according to [1, 2]. For more clearness figure 3 and table 1 are shown.
Figure 2: Steel temperature based on ISO-fire in function of the section factor ($50 \text{m}^{-1} \leq F \leq 300 \text{m}^{-1}$)

Figure 3: Steel temperature based on ISO-fire in function of the section factor ($50 \text{m}^{-1} \leq F \leq 300 \text{m}^{-1}$)

Table 1: Steel temperature based on ISO-fire in function of the section factor
Section factor is the relation between the fire exposed area \( (A_s) \) and the heated volume \( (V) \) of the structural steel member. For prismatic bars, section factor can be expressed, also, as the relation between the fire exposed perimeter and the cross section area of the member (exp. 14).

\[
F = \frac{u}{A_s}
\]  

(14)

In the derivation of exp. 1, it was considered that the steel member reaches equal and evenly distributed temperatures at the surface as well as in the volume. In different situations, for instance, the case of members belonging to compartment’s seal, this expression can be used, but, one must determine \( F \) in relation to the part of the volume that can be admitted with even temperature distribution that is equal to that of the fire exposed surface or calculate the temperature by more precise methods of thermal analysis. Table 2, extracted from [1, 2], supplies expressions for calculation of section factor of sections usually found in civil construction.

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Table 2: Section factor for unprotected steel members.
(1) Open section exposed to fire on all sides:
\[ F = \frac{\text{perimeter}}{\text{cross section area}} \]

(2) Tube exposed to fire on all sides:
\[ F = \frac{d}{t (d - t)} \]

(3) Open section exposed to fire on three sides:
\[ F = \frac{\text{perimeter exposed fire}}{\text{cross section area}} \]

(4) Hollow section (or welded box section of uniform thickness) exposed to fire on all sides:
\[ F = \frac{b + d}{t (b + d - 2t)} \]

(5) I-section flange exposed to fire on three sides:
\[ F = \frac{b + 2t_f}{bt_f} \]

(6) Box section exposed to fire on all sides:
\[ F = \frac{2(b + d)}{\text{cross section area}} \]

(7) Angle exposed to fire on all sides:
\[ F = \frac{2}{t} \]

(8) I-section with box reinforcement, exposed to fire on all sides:
\[ F = \frac{2(b + d)}{\text{cross section area}} \]

(9) I-section with box reinforcement, exposed to fire on all sides:
\[ F = \frac{2(b + t)}{bt} \]

(10) Flat bar exposed to fire on three sides:
\[ F = \frac{b + 2t}{bt} \]
Note that sections exposed in only three sides constitute exceptions to the explained by this work (exp. 1). Nevertheless, for those cases, the Standards admit that the contact in the fourth, besides thermally protecting the surface of contact, reduces the average temperature in member’s volume leading to results in the safe side. For case (10) of the table 2, it is recommended to take for section factor the relation between the fire exposed perimeter and the plate’s area. To evaluate the degree of approximation of this recommendation, this case was modeled as fig. 4, adopting a 10 cm thick x 60 cm wide slab and a 1.25 cm thick x 20 cm wide plate.

The model was divided in 0.01 mm x 0.01 mm square finite elements. With the aid of the thermal analysis software STC – Super Tempcalc [7], the structural assembly was submitted to the standard curve of temperature rise till 30 min. The found field of temperatures is shown in figure 5.

In figs. 6 to 8 the model isotherm lines after 30 min heating are presented
Applying exp. 1, it is verified that, for a section factor equal to 90 m$^{-1}$, it is found $\theta = 734^\circ$C. On the other hand, as seen in figures 8 and 9, the actual temperature varies between 570°C (most part) and 620°C. In order that exp. 1 results in $\theta = 590^\circ$C (admitted as the average in the plate) a section factor equal to 67 m$^{-1}$ would be needed. This demonstrates that, despite the method of exp. 1 not being applicable to this case, the standards recommendation is on the safe side. Similar thermal analysis was performed for 90 min. Using exp. 1, it is found $\theta = 1000^\circ$C. According to thermal analysis via STC, the temperature varies from $\theta = 940^\circ$C (most part) to $\theta = 950^\circ$C (figure 6). To reach these temperatures by means of exp. 2.1 the section factor should be around 27 m$^{-1}$. As can be observed, there is no clear relationship between section factor and temperature if the adopted hypotheses for derivation of exp. 1 are not respected. However, the perimeter reduction simplification is, again, on the safe side.
With this example it is verified that the calculated temperature using the section factor recommended by Table 2, case (10), leads to safe results. Intuitively, one can extend this conclusion to case (5). It is not what occurs in case (3). Nevertheless, there are works that indicate that the calculated temperature by means of exp. 1 is close to the cross section maximum temperature experimentally determined [8].

In consequence, for situations presented by Fig. 10, if the masonry is not a compartmentation wall, i.e., there is a chance of fire occurrence in both sides of the wall, the perimeter will be the total perimeter deducted by the part in contact with the wall. The area to be considered, however, will depend on the degree of protection of the wall or slab, but is a good approximation to use the total area.

In the situation of Fig. 10b, if masonry is a compartmentation wall, with thickness equal to the web height, the perimeter is the one presented by the figure and, in general, the recommended area of Table 2 (5) is adopted.

Section factor for cases (5), (9) and (10), when plate thickness can be overlooked in relation to its width, can be calculated by exp. 15.

\[ F = \frac{1}{t} \]  

In other cases usually found in civil construction (Fig. 10), application of exp. 1 is not feasible unless a more accurate thermal and structural analysis is performed. When the wall is compartmenting, for situations represented in 10a, 10b e 10c, there is no specific way for the calculation of F. The perimeter to be considered is the one highlighted in the figure. The difficulty resides in the definition of the heated area. The situation shown in figure 10d is impossible to be calculated by means of the simplified method presented by this work. A severe fire at one of the sides can reduce the strength and the modulus of elasticity of one of the box shaped beam’s webs, turning the whole set unstable because requiring a tube shape to resist the forces.
By means of these simple examples, it’s intended to show that exp. 1, yet very much used, does not solve several cases frequently found in civil construction. There is the need of thermal analysis works to solve them and, if feasible, to find expedite means for dimensioning. Similar conclusion is possible to reach, when expressions, like exp. 16 [2] or exp. 17 [9], to determine the temperature of protected steel, is used.

\[
\begin{align*}
\theta_a(t + \Delta t) - \theta_a(t) &= \frac{F}{t_m/\lambda_m} \left( \frac{\theta_g(t) - \theta_a(t)}{\rho_a c_a \left(1 + \Phi/3\right)} \right) \Delta t \\
&\quad - \left[ \theta_g(t + \Delta t) - \theta_g(t) \right] \left( \Phi/e^{10} - 1 \right) \\
&= \frac{F}{t_m/\lambda_m} \left( \frac{\theta_g(t) - \theta_a(t)}{\rho_a c_a \left(1 + \Phi/4\right)} \right) \Delta t \\
&\quad - \frac{\theta_g(t + \Delta t) - \theta_g(t)}{\Phi + 1}
\end{align*}
\]

(18)

(19)

4 Conclusion

The expression for calculation of temperature in thermally unprotected structural members was derived within this work. It was demonstrated that this expression could only be employed in the case of slender members, just as steel, and not in contact with heat-sink elements, like slabs and masonry. A case, despite not obeying the adopted hypotheses for the method’s derivation was thermally analyzed, with the aid of Super Tempcalc computer software, and it was found to be a simplification on the safe side, allowing the use of exp. 1. For other situations, frequently found in civil construction, further works involving thermal and structural analysis are extremely needed.
References


Nomenclature

A - area (m²)
A_a - area of the steel member exposed surface (m²)
A_s - area of the cross section (m²)
F - section factor, the ratio between the exposed surface area and the volume of the steel, (m⁻¹)

\dot{Q}_{abs} - heat absorbed by the steel member (W)
\dot{Q}_c - convective heat flux (W)
\dot{Q}_d - design value of the heat flux (W)
\dot{Q}_k - characteristic value of the heat flux (W)

\dot{h}_r - radiant heat flux (W)
V - volume of the steel member (m³)
c - steel specific heat (J/kg °C)
m - mass of the steel member (kg)

\dot{h} = \dot{h}_c + \dot{h}_r - heat flux (convective and radiant) to unity surface area. (W/m²)
\dot{h}_c - convective heat flux to unity surface area. (W/m²)
\dot{h}_d - design value of the heat flux to unity surface area. (W/m²)
\dot{h}_k - characteristic value of the heat flux to unity surface area. (W/m²)
\( \dot{h}_r \) - radiant heat flux to unity surface area. (W/m\(^2\))

\( t \) - thickness (m)

\( u \) - perimeter (m)

\( \alpha = \alpha_c + \alpha_r \) - coefficient of heat (convective and radiant) transfer (W/m\(^2\) °C)

\( \alpha_c \) - coefficient of heat transfer by convection (W/m\(^2\) °C)

\( \alpha_r \) - coefficient of heat transfer by radiation (W/m\(^2\) °C)

\( \varepsilon \) - emissivity

\( \varepsilon_{1,2} \) - surface emissivity of the bodies 1 and 2 (-)

\( \varepsilon_r \) - surface emissivity of the structural member

\( \sigma \) - Stephan Boltzmann constant = \( 5.6 \times 10^{-8} \) W/m\(^2\) °C

\( \theta \) - temperature (°C)

\( \theta_a \) - average temperature of the steel surface exposed to fire (°C)

\( \theta_{a,vol} \) - average temperature in the volume (°C)

\( \dot{\theta}_{a,vol} \) - variation of the volume temperature per unit time (°C/s)

\( \theta_g \) - hot gases' temperature (°C)

\( \theta_1 \) e \( \theta_2 \) - temperature of the bodies 1 and 2 (°C)

\( \Delta \theta_a \) - variation of the steel temperature

\( \rho \) - steel density (kg/m\(^3\))