Latin American Journal of Solids and Structures
www.lajss.org

A Simplified Method to Estimate the Fundamental Frequency of Skew Continuous Multicell Box-Girder Bridges

Abstract

Here Fundamental Frequency is the main factor used to calculate the vibration of a vehicle when passing across a bridge. With knowledge of the fundamental frequency, bridges can be evaluated and designed in such a manner so as to avoid the critical range of 1.5 Hz to 4.5 Hz, which is occupied by most vehicles. Therefore it is crucial to develop a reliable method to estimate the fundamental frequency of bridges. To overcome the above issue, numerical analysis combined with a theoretical method is applied to estimate the fundamental frequency of multicell box-girder bridges. The effect of span-length, number of boxes and skew angle on the estimation of this factor is discussed. Finally, reliable expressions are proposed to predict the first fundamental frequency of this type of bridge, and the accuracy of the expressions is verified. The results indicate that the fundamental frequency decreases when span length increases, due to development of crack as well as decrease stiffness of girders.

Keywords

Fundamental frequency, skew angle, box-girder bridges

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1 INTRODUCTION

The dynamic effects on a bridge may have a considerable influence on its ultimate limit state behavior by amplifying the maximum stress experienced by each of its members. The dynamic responses of a bridge are influenced by several factors. In order to evaluate these factors for determining the dynamic responses it is necessary to characterize the bridge in term of its fundamental frequency and mode shapes.

Traditionally bridges are designed using static loads, which are increased by the dynamic load allowance (DLA) factor of the bridge (Ashebo et al., 2007a,b; Billing, 1984;Chang and Lee, 1994; Kashif, 1992). Extensive research and development has been carried out to understand the vibration of bridges as a result of natural sources of vibration, and to determine the dynamic allowance factor as a function of the fundamental frequency due to its uniqueness (OHBDC, 1983; Samaan et al., 2007; Sennah et al., 2004; Senthilvasan et al., 2002; Zhang et al., 2003). Meanwhile, in practice heavy trucks establish a quite narrow range of frequencies, 1.5 Hz to 4.5 Hz, therefore it is important to find a reliable method to estimate the fundamental frequency of bridges and design structures in such a way as to avoid this critical range of frequencies (Moghimi and Ronagh, 2008). In addition, due to vibration, the dynamic deflection can cause discomfort to pedestrians using the bridge. It has been known that the human body tends to react more to torsional oscillations than flexural ones.

A large number of studies on free-vibration analysis have been performed on box-girder bridges. Komatsu and Nakai (1970) and Sennah(1998) evaluated the natural vibration responses of straight and curved I- or box-girder bridges using Vlasov's beam theory. Subsequently, Heins and Sahin (1979) applied finite difference methods to solve the differential equations of motion based on Vlasov's thin wall beam theory.

Culver and Oestel (1969) used a close form solution for the equation of motion to determine the fundamental frequencies of horizontal curved beams. Cheung and Cheung (1972)used the finite strip technique to evaluate the free-vibration analysis of straight and curved bridges.
Cantieni (1984) conducted an experimental study to establish a reliable relationship between the fundamental frequency and maximum span length of bridges. It was found that the bracing system significantly affected the fundamental frequency of composite bridges. Finite element analysis was used to extract the dynamic characteristics of bridge-vehicle interaction and establish reliable expressions for predicting the dynamic responses of curved multiple box-girder bridges, but the recommendations are unacceptable in the case of skew bridges.

A number of studies used the finite element method to examine the forced vibration response of instrumented passing vehicles and free-vibration responses of composite cellular box-shaped bridges. Comparison of the studies indicated that the finite element method obtained sufficiently reliable results, compared with other numerical methods (Brownjohn et al., 2008; Fujino et al., 2010; Lin and Yang, 2005; Siringoringo and Fujino, 2012). In order to reduce the significant difference between the estimation of the fundamental frequency obtained from the codes and theoretical methods, Gao et al.(2012) proposed a numerical improved method for straight bridges. However, some limitations still exist for these methods to actually be used in practice. Some enhancements, improvements or further research is required to allow them to be applied in the field (Wang et al., 2013). The results of a study on effects of damage and decay ratio on fundamental frequency by Pandey and Benipal (2011) indicated that that even SDOF bilinear beams have multiple resonance frequencies at which the resonance can take place. However the actual structure is never a SDOF and may have infinite degree of freedom, hence multiple fundamental frequencies depending upon the actual stiffness operating based on cracking pattern.
According to the above, the present studies have mainly concentrated on the free-vibration analysis of straight and curved bridges, and there are no reliable methods to determine the fundamental frequencies of skewed bridges. Therefore, in this study the results of an extensive numerical study on the free-vibration feature of continuous skewed concrete multicell box-girder bridges are evaluated. The prototype bridges are analyzed by using a three-dimensional finite element method. The empirical expressions are established using regression analysis, to determine the fundamental frequencies of such bridges.

2 PRESCRIPTION OF ANALYTICAL BRIDGES AND MODELING

The prototype bridge used in this study is highly representative of the majority of concrete skew multicell box-girder bridges. Eighty-five typical bridges with a span length ranging from 30 to 90 m have been designed based on the Canadian Highway Bridge Design Code(CHBDC, 2006). The number of boxes varies from two to six, dependent on the width of the bridge. All selected bridges are two-equal-span continuous, with bridge widths (W) of 9.14 m, 14.00 m and 17.00 m. Since most heavily vehicle frequencies occupy a relatively narrow frequency band in practice, 1.5 To 4.5, it is preferred to design bridge in such a manner as to avoid this critical range, if at all possible.

The preliminary study indicated that the thickness of the deck has an insignificant effect on the dynamic response of multicell box-girder bridges, so constant values of 20cm and 15 cm were selected for the upper and bottom deck thicknesses, respectively. To consider the effect of skewness on the free-vibration response of this type of bridge, the skew angle was ranged from 0 to 60°, which is within the range of applicability introduced by the American Association of State Highway and Transportation Officials' Load Resistance Factor Design(AASHTO, 2008). Table 1 shows the characteristics of the selected bridges, in which the symbols N_B and N_L stand for the number of boxes and number of loaded lanes, respectively. A typical bridge cross-section is shown in Figure 1. Reinforced concrete intermediate diaphragms are used for all prototype bridges at spacing of 7.5 m (25 ft), alongside the end diaphragms to enhance the stability of the structure under construction loads.

In this study, the prototype bridges are modeled with CSIBRIDGE software V15, using a four node, three-dimensional shell element with six degrees of freedom at each node. The top and bottom shell elements of the webs are integrated with the top and bottom slabs at connection points to improve the compatibility of the deformations (Mohseni and Khalim Rashid, 2013). The bridge modeling was verified by comparing the live load distribution factor (LDF) derived from field testing, and those from the method adopted herein. Boundary conditions are simulated using hinge-bearing at the starting abutment, and roller-bearings for all other supports. Figure 2 shows a finite element model of a 200 m three-box multicell box-girder bridge.

Set	L (m)	N_B	N_L	W	d'	d''	t_w	Skew (ϑ)	L_c (m)
1	30, 45, 60, 75, 90	2,3	1,2	9.10	0.20	0.15	0.10	0,15,30,45,60	0.610
2	30, 45, 60, 75, 90	2,3,4	2,3	14.0	0.20	0.15	0.10	0,15,30,45,60	1.20
3	30, 45, 60, 75, 90	3,4,5,6	2,3,4	17.0	0.20	0.15	0.10	0,15,30,45,60	1.45

Table 1: Parameters considered in parametric study (in meters).

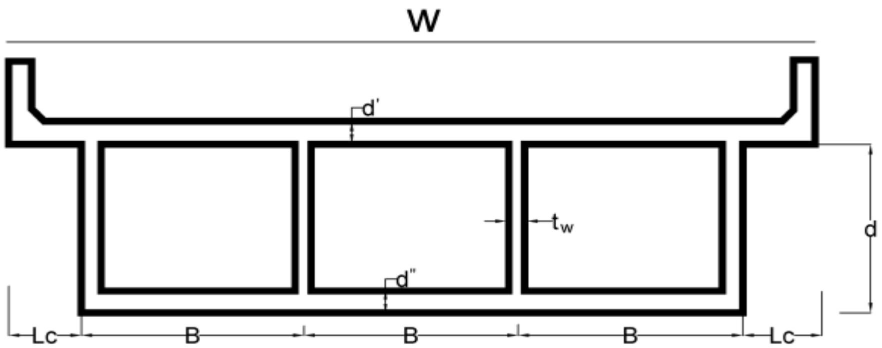

Figure 1: Typical cross-section.

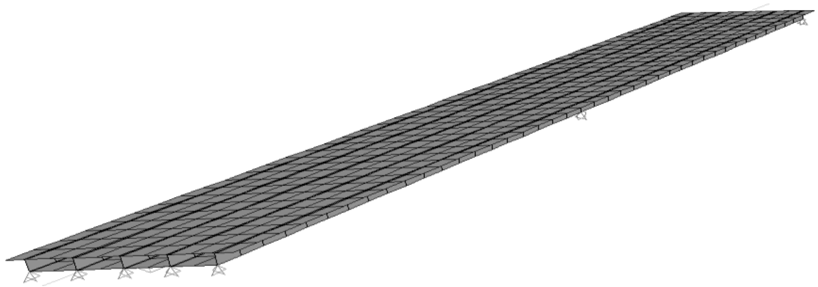

Figure 2: Finite element of a skew four-box bridge.

3 RESULTS AND DISCUSSION

A sensitivity study is first performed to investigate the influence of the key parameters on the fundamental frequency of the prototype bridges. These parameters consisted of the span length, number of lanes loaded, skew angle, and number of boxes.

3.1 Effect of span length

The effect of span length on the fundamental frequency for three-box bridges with span lengths ranging from 30 m to 90 m is plotted in Figure 4. It can be observed that the fundamental frequency decreases considerably with increasing span length of bridge, within a maximum range of 62.5%. This difference is caused by the influence of other parameters, especially the skew angle. The mode shapes for a three-lane, three-box bridges with a span length of 45 m are plotted in Figure 3. As expected, for straight bridges the first mode shapes a real ways purely flexural (Ashebo et al., 2007a,b), while torsional effects contribute to the first mode shapes of concrete skew bridges as indicated in Figure 5. The high drop in fundamental frequency when span length increases is due to development of cracks and subsequently, decreases in modal parameters such as damping ratio and stiffness of girders. The same observation was obtained by WU et al. (2011) from experimental studies on reinforced slab and concrete beam.

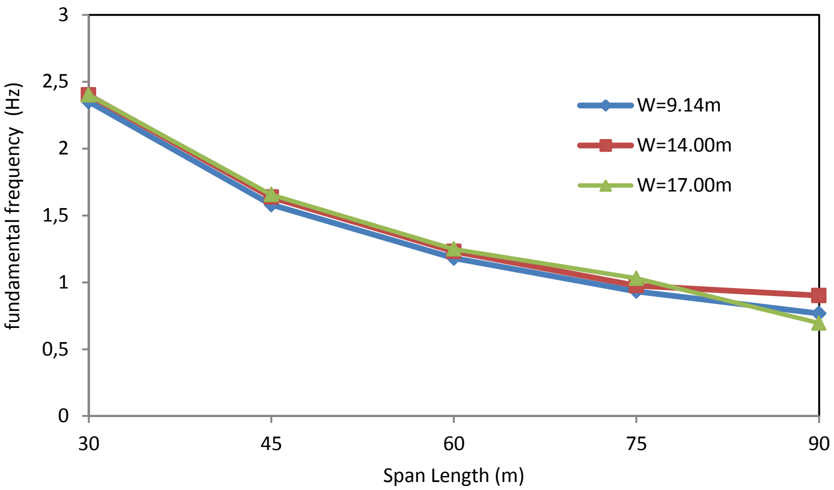

Figure 3: Effect of span length on fundamental frequency for three-box bridges.

3.2 Effect of Number of Lanes

Three-box 30m to 90m straight bridges were evaluated to illustrate the effect of the number of loaded lanes on the fundamental frequency, as indicated in Figure 4. It is revealed that the number of lanes only slightly increases the fundamental frequencies of multicell box-girder bridges. An enhancement in the fundamental frequency of 60 m bridges of about 5.5% was obtained when the number of lane loaded increased from 2 to 4, and increased by approximately 1.5% in the case of the bridge with a 30m span length. Therefore it is concluded that the number of lanes loaded as well as truck positions have no significant effect on the fundamental frequency. The same results were obtained for skew bridges, but due to space limitations are not show herein.

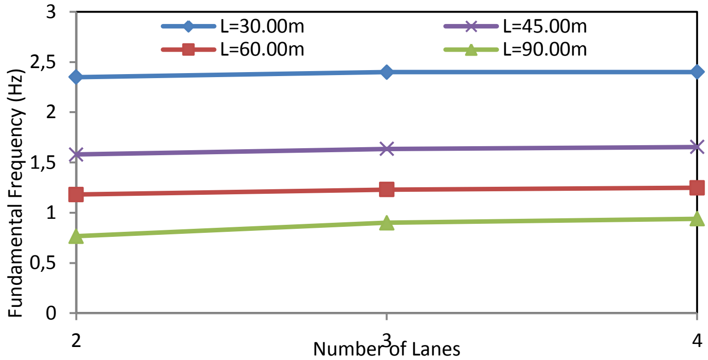

Figure 4: Effect of number of lanes on fundamental frequency.

3.3 Effect of Number of Boxes

The effect of the number of boxes on the fundamental frequency is considered in Figure 5. Three-lane straight bridges with span lengths of 30 m, 45 m and 90 m, and with various numbers of boxes are analyzed. The graphs shown reveal that the increase in the number of boxes only slightly decreases the fundamental frequencies. A decrease in the fundamental frequency of a 30 m bridge of up to 3% is observed as the number of boxes increases from 2 to 5, and declinesbyapproximately6% and 13%for bridges with span lengths of 45 m and 90 m, respectively. It can be concluded that the effect is not crucial because of the fact that the torsional and bending stiffnesses of the close-box shape structure do not obtain a significant effect from the increase in the number of webs (Mohseni and Khalim Rashid, 2013). The results, not presented herein, indicated that the number of boxes have no influence on the mode shapes of multicell box-girder bridges.

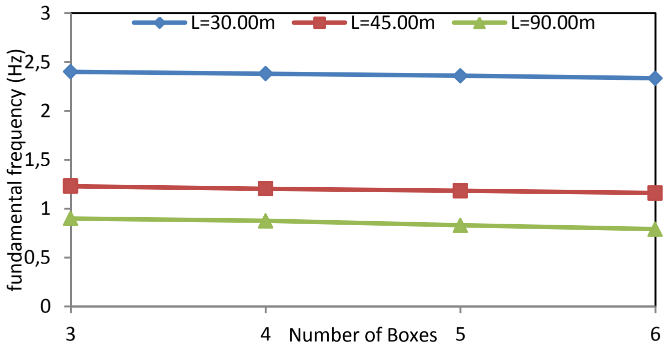

Figure 5: Effect of number of boxes on fundamental frequency.

3.4 Effect of Skew Angle

The relationship between the fundamental frequency and skew angle of three-box 60m bridges are shown in Figure 6. The results illustrate that the skew angle has a considerable influence on the fundamental frequency, in that it increases rapidly with an increase in the skewness of the bridge. For example, the fundamental frequency increases by almost 42% for bridges with different numbers of loaded lanes, when skew angle was varied from 0 to 60º.

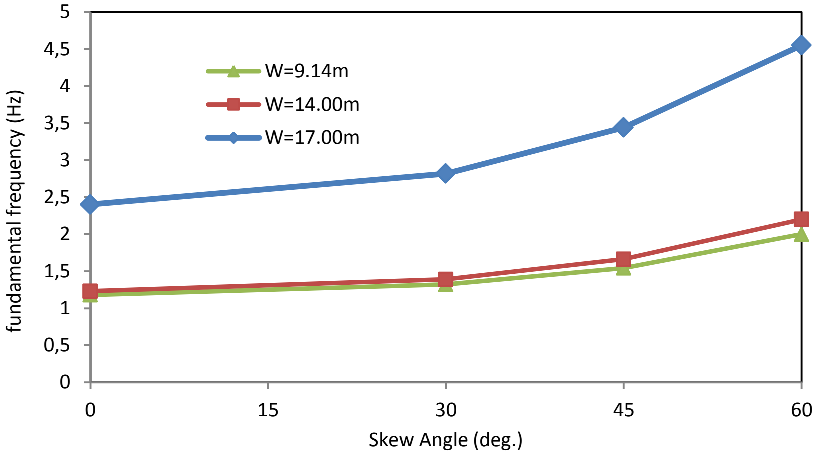

Figure 6: Effect of skew angle on fundamental frequency.

The first mode shapes of a 200m four-box bridge with different skew angles were analyzed for the effect of skew on the dynamic behavior of the bridges, and the results are presented in Figure7. It was found that for bridges with a skew angle of 0 to 45°, the mode shapes and fundamental frequencies are entirely flexural, regardless of the length of the span, while a torsional mode governs for bridges with a skew angle of 60°. Therefore it was concluded that the modal analysis of multicell box-girder bridges is similar to a beam, in that the beam response dominates the dynamic behavior of the bridges. This is due the fact that the span of the bridge is far greater than the sectional width. Therefore it is concluded the skew angle should be taken into account when developing the proposed equations for skew multicell box-girder bridges.

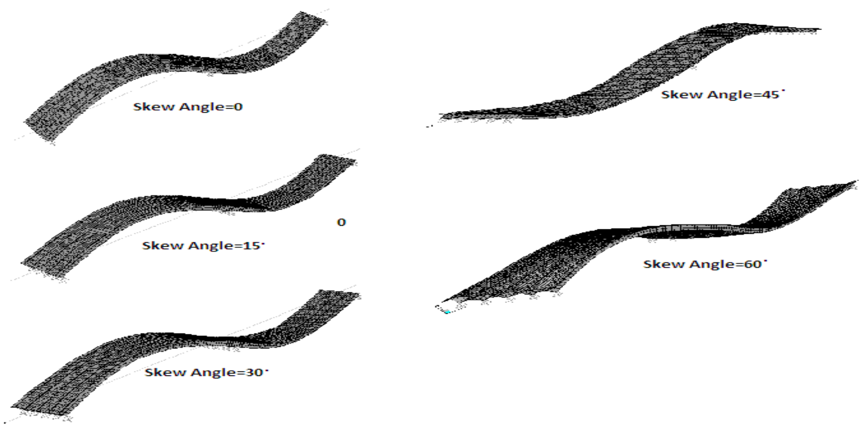

Figure 7: Mode shapes of four-box bridges with different skew angles.

4 EMPIRICAL EQUATIONS FOR FUNDAMENTAL FREQUENCY

A statistical analysis is performed on the findings from the parametric study of straight multicell box-girder bridges, to derive an expression for predicting the fundamental frequency, f_s, as follows:

$$f_s = \frac{70.14}{L(m)} = \frac{234.2}{L(ft)} \tag{1}$$

where L is the centerline length of the skew bridge over one span in meters. The precision of this expression is around 3% for all the straight bridges analyzed in this study, regardless of the number of boxes and lanes loaded. Furthermore, Eq. (1) also offered reasonable results for simply-supported straight bridges.

In order to take into account the considerable effect of skewness, a modification factor is developed for multiplication with Eq. (1). The modified expression for the fundamental frequency of the prototype skew bridges, f_c, is introduced as:

$$f_c = \mu . f_s \tag{2}$$

where μ is a modification factor obtained as:

$$\mu = (1.590 + 0.0156 \times N_B) \times \frac{1}{(Cos\theta)^{0.82}} \tag{3}$$

It should be noted that the number of boxes does to some degree affect the fundamental frequency of skewed bridges, as considered in Eq. (3). The proposed equation is relatively precise for determining the fundamental frequency for the two-span skew multicell box-girder bridges used in the parametric study with an error of up to 3%. The statistical values for the ratio between the proposed Eq. (3) and finite element results (Equation/FEM) are determined for verification purposes. The standard deviation (SD) of 0.036 revealed that the first fundamental frequency obtained from the equation is sufficiently close to the mean values. The average value (AVG) of 1.008 and coefficient of variation (COV) revealed an excellent match between the analytical and numerical methods.

In Figure8, the relationship between the first fundamental frequency of the skew multicell box-girder bridges obtained from the finite element method and proposed equation is shown. The coefficient of determination R^2 is 0.993, slightly lower than unity, which is excellent and reveals a low variation.

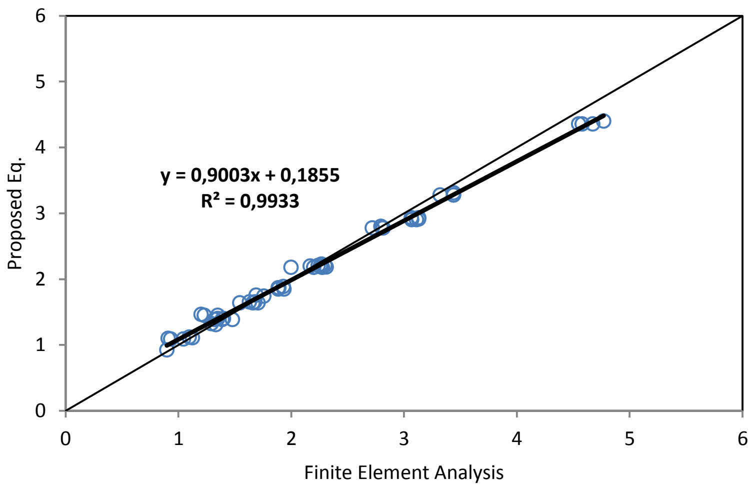

Figure 8: Fundamental frequency derived from FEM versus proposed equations.

5 SUMMARY OF AVAILABLE METHOD

As mentioned previously, there is no reliable analytical method or field test for the fundamental frequencies of concrete multicell box-girder bridges. Bridge engineers often apply the proposed equations to railway bridges in order to obtain the fundamental frequency and dynamic load allowance of highway bridges under vehicle loads (Frýba, 1996). Therefore a comparative study is performed to evaluate the accuracy of the available equations and to verify the proposed expressions (Eqs. (1) to (3)).

The proposed equations for fundamental frequency are compared with the available empirical formulae suggested by Frýba and Frýba (1999), the International Union of Railways (UIC, 1979), Samaan et al.(2007), and the British Standard Institution (BS EN 2003). Frýba (1996) proposed that the fundamental frequency for railway bridges of all types, materials and structural systems can be obtained from:

$$f_1 = \frac{133}{L^{0.90}} \quad (in\, SI) \tag{4}$$

and the International Union of Railways developed the fundamental frequency as follows:

$$f_1 = \frac{208}{\mathrm{L(m)}} \tag{5}$$

These equations were developed through sensitivity statistical evaluations on a large number of field tests on real railway bridges. The British Standard Institution (BS EN 2003) suggested that the first fundamental frequency of bridges of all types and materials should be within the following range:

i) Lower bound (for span lengths of 20 m ≤L(m) ≤100 m)

$$f1 = \frac{23.58}{L^{0.592}} \quad \text{(in SI)} \tag{6}$$

ii) Upper bound (for span lengths of100 m ≤L(m))

$$f1 = \frac{94.76}{L^{0.745}} \quad \text{(in SI)} \tag{7}$$

In addition, Samaan et al. (2007) recommended an empirical equation for the fundamental frequency of horizontally-straight composite multiplied box-girder bridges as follows:

$$f_1 = \frac{94}{L(m)} \tag{8}$$

The comparison between the available methods and the proposed expressions (Eq. (1) and Eq. (2)) is shown in Figure 9. For simplicity, only the trend line of the equations is plotted. From the graphs it can be seen that the available equations estimate highly conservative values for the first fundamental frequency of multicell box-girder bridges. That is due to the fact that except Eq. (8), the other equations are derived for railway I-girder bridges of high-strength material and narrow width. Meanwhile, the work carried out by Samaan et al. (2007) only covered straight and curved bridges, and skew bridges are not considered.

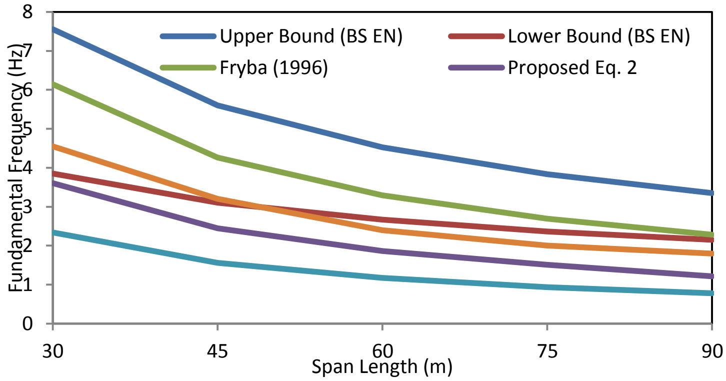

Figure 9: Comparison of first fundamental frequency from various methods.

6 CONCLUSION

An extensive study was conducted using three-dimensional finite element analysis to evaluate the effect of major parameters on the fundamental frequency of continuous concrete skew multicell box-girder bridges. It was evident that the fundamental frequency decreases as the span length of the bridge increases, and increases as the skew angle on the support line and piers increases. The empirical expressions were derived for the first fundamental frequency using statistical analysis. For convenience when designing, the expressions are in terms of the span length of the bridge, and the effect of skewness is taken into account by a skew correction factor in the proposed equation. A comparison of the values derived herein with those given in current bridge codes indicated that latter often estimate highly conservative values for the first fundamental frequency of skewed multicell box-girder bridges.

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