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Modelling and non-linear analysis of beam-elements in reinforced concrete Vierendeel sandwich plates considering nodal stiffness domain effects

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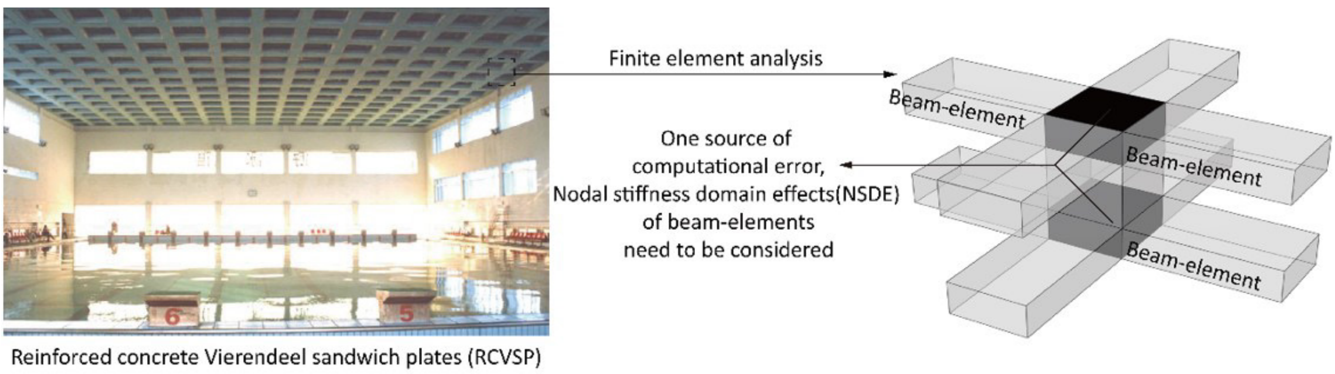
Abstract

The results of finite element analysis of reinforced concrete Vierendeel sandwich plates (RCVSP) using beam-plate elements calculation model show a significant discrepancy with the actual results. Considering the structural characteristics of RCVSP and the calculation method of beam-elements, the assumption of the existence of nodal stiffness domain effects (NSDE) is proposed for its beam-plate elements calculation model. In order to verify this assumption, the existing experimental data of RCVSP is used to compare with the finite element analysis results of the beam-plate elements calculation model. The analysis results show that there is NSDE in the finite element analysis of RCVSP using beam-plate elements calculation model. The beam-elements calculation model FeaR1, which fully considers NSDE, can effectively restore the calculated stiffness lost. Compared with the solid-elements calculation model of RCVSP, the calculation model FeaR1 is more convenient for modeling, enhances computational efficiency, achieves the stiffness restoration rate of approximately 80%, and maintains the average error of no more than 16%.

Keywords

Vierendeel sandwich plates, reinforced concrete, nodal stiffness domain effects, beam-elements, Stiffness restoration

Graphical Abstract



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1 INTRODUCTION

Academician Ma Kejian's team (Ma et al., 2006) proposed a Vierendeel sandwich plates (VSP) floor system in 1995. This system consists of upper and lower ribbed beams, shear keys and panels (Figure 1), and is commonly utilized in large-span buildings because of its benefits, such as large span, light deadweight and good stiffness. It is often constructed with I-beams and concrete plates or reinforced concrete, with the maximum span of 39 meters (Ma et al., 2008). VSP structure has a wide range of applications. Currently, it is not only used for floor slabs but also being researched for potential use in bridges (Yu et al., 2022).

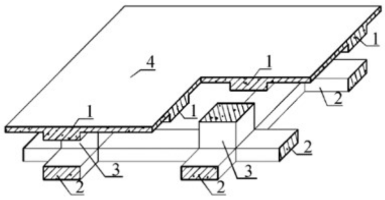

Figure 1. *Constitution of VSP*

Due to the complex structure of VSP, engineering units find it challenging to perform fast and accurate calculations and analyses. Therefore, many experts have studied the calculation method of VSP. Xiao et al. (2000) and Zhang et al. (2006) proposed the method of iso-substitution stiffness under static force for VSP. This method simplifies VSP into a cross-beam structure based on the principle of flexural stiffness equivalence, and internal forces and deformations are calculated by using structural analysis software. Some Chinese local Codes for VSP (China, 2005; 2014) use this analysis method. Due to the neglect of shear stiffness effects, the accuracy of the calculations is compromised. Ma et al. (2008) derived the tenth-order partial differential equation by using the continuum method under consideration of effect of shear stiffness, and proposed several analytical solutions under specific conditions. Due to the complexity of solving under usual conditions, it is primarily utilized for scientific research rather than engineering design. Liu et al. (2023) simplified on the continuum method and solved equations to derive parameters of VSP using the variational method. However, this method is difficult to apply to plates with unusual shapes and cannot be used to calculate the internal forces of the member specifically, limiting its role in actual projects.

There are currently three mainstream finite element analysis (FEA) modeling methods: Computational modeling of beam-plate elements considering second-order shear (Yong et al., 2001; Chen et al., 2001): The upper and lower ribbed beams and shear keys are modeled as beam-elements, and the panels are modeled as plate-elements. The advantage of this model is its simplicity, but it often results in a significant error between calculated and actual results. The solid-beam-plate elements calculation model (Wei et al., 2017; Chen et al., 2018): Compared with the first model, shear keys with complex force is modeled by solid-elements instead of beam-elements .This method often requires consideration of the nodal coupling effect at beam bond connection, modeling is more complex, not suitable for engineering applications. The third is a computational model consisting only of solid-elements (Yao et al., 2017): This method offers high computational accuracy, but the discrete construction brings many nodes with a huge number of degrees of freedom, which aggravates the computational scale and makes it challenging to implement in practical engineering applications. Sheng et al. (2023) proposed the use of the superelement method to coalesce degrees of freedom to reduce computational scale, but there are still more ways of cohesive selection of substructure degrees of freedom for RCVSP, which are more challenging to model.

In order to make the RCVSP structure widely selected in engineering, the simple beam-plate elements calculation model is the most suitable for the engineering needs (Sadeghi et al., 2020; Parente et al., 2023), which is also the mainstream structural design software on market for most of structures. In the study of beam-elements, scholars (Ali and Sridharan, 2005; Rezaee Hajidehi et al., 2018; Zhao et al., 2019; Tang et al., 2020; Talele et al., 2021) often propose computational methods that are tailored to the specific requirements of the target structure. Yang et al. (2017) proposed a new variable-length beam unit that considers the effect of length and can effectively characterize the dynamics of beams with a circular cross-section; Lima and Faria (2017) proposed a new C1 beam element based on Overhauser interpolation, which is used to solve the beam-to-frame issue; Calisto et al. (2023) presented a numerical model of a hybrid steel beam with an I-shaped cross-section, showcasing the exceptional bending resistance of the hybrid beam; Kim et al. (2024) developed a beam cell for nonlinear analysis of three-dimensional reinforced concrete frames subjected to fire; Xiang and Gao (2024) proposed a convergence-enhanced Timoshenko beam element considering finite deformation for stochastic nonlinear analysis of reinforced concrete members.The main research idea of the scholars is to enhance the calculation method of the beam elements, beginning with the target structural force, to align it more effectively with the structural requirements (Cambronero-Barrientos et al., 2017; Hansen and Jönsson, 2019;

Cambronero-Barrientos et al., 2022; Zhao et al., 2022; Chiu and Yu, 2023). This leads to direction of research that RCVSP needs a more tailored way of beam-elements calculation.

Therefore, according to the characteristics of RCVSP, and considering the calculation method of traditional beam-elements, the assumption of beam-elements calculation model is proposed: There is a nodal stiffness domain effects (NSDE) in the traditional model for calculating RCVSP. Utilizing the available experimental data, the proposed assumption of NSDE is analyzed and validated through the use of the commercial structural design software MidasGen and the commercial FEA software Ansys. Finally, the analyzed results are used to find a beam-elements calculation model more suitable for RCVSP.

2 Theory Analysis

2.1 Nodal error analysis of beam-elements calculation model

In finite element analysis (FEA), the use of beam-elements to simulate the beams and columns of a target building is a more common approach. In contrast to solid-elements, beam-elements reduce the number of degrees of freedom in the overall calculation. The advantages in terms of computational efficiency and computational requirements are particularly evident in the simulation of large or tall structures. However, the reduction in the number of degrees of freedom also reduces the overall accuracy of the calculation, which is not as precise as that of solid-elements. From the Euler beam-elements to the Timoshenko beam-elements and beyond, the evolution of calculation methods and the development direction of beam-elements have been inclined towards considering a more comprehensive force scenario to enhance result accuracy. However, in the majority of engineering applications, such errors fall within an acceptable range. At present, most engineering design software on the market still utilizes beam-elements for calculations, in conjunction with safety factors specified in engineering production regulations, such as China's PKPM, MidasGen, and others.

At present, the results obtained by the beam-elements calculation method for members are quite accurate. However, the calculation error of beam-elements at the nodes is difficult to improve (Karttunen et al., 2016; Tai and Chan, 2016). For example, when extracting the stiffness matrix of a planar beam-element (Eq. 1), it can be clearly seen that one of the parameters that has a greater influence on the stiffness of beam-elements: length of beam-elements *L* is inversely correlated with the stiffness of beam-elements (Davis et al., 1972).

$$\begin{Bmatrix} f_{xi} \\ f_{yi} \\ M_{zi} \\ f_{xj} \\ f_{yj} \\ M_{zj} \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_z}{L^3(1+\Phi)} & \frac{6EI_z}{L^2(1+\Phi)} & 0 & -\frac{12EI_z}{L^3(1+\Phi)} & \frac{6EI_z}{L^2(1+\Phi)} \\ 0 & \frac{6EI_z}{L^2(1+\Phi)} & \frac{(4+\Phi)EI_z}{L(1+\Phi)} & 0 & -\frac{6EI_z}{L^2(1+\Phi)} & \frac{(2-\Phi)EI_z}{L(1+\Phi)} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3(1+\Phi)} & -\frac{6EI_z}{L^2(1+\Phi)} & 0 & \frac{12EI_z}{L^3(1+\Phi)} & -\frac{6EI_z}{L^2(1+\Phi)} \\ 0 & \frac{6EI_z}{L^2(1+\Phi)} & \frac{(2-\Phi)EI_z}{L(1+\Phi)} & 0 & -\frac{6EI_z}{L^2(1+\Phi)} & \frac{(4+\Phi)EI_z}{L(1+\Phi)} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ \theta_{zi} \\ u_j \\ v_j \\ \theta_{zj} \end{Bmatrix} \tag{1}$$

Where,

$$I_z = \frac{bh^3}{12}; \quad \Phi = \frac{12EI_z}{GA_sL^2}; \quad A_s = \frac{5}{6}bh \tag{2}$$

To calculate length of beam-elements, planar beam-elements is used as shown in Eq.3:

$$L = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \tag{3}$$

This means that the length of beam-elements is related to the position of the end nodes of the beam-elements. This calculation method is accurate for single or continuous beam-elements. In the case of T-jointed beams, the overlap of the beams is visible at the intersection of the nodes (Figure 2.a). Such overlapping regions are irrelevant in the calculation of beam-elements. However, for a more accurate simulation, it is important to note that the calculated lengths of some beam-elements are simply overestimated. For instance, in the section of Figure 2.a where the yellow color overlaps, the length of this portion should not be included in the calculation of the total length of the yellow beam. This overestimation is primarily reflected in the miscalculation of the total self-weight of the structure. Assuming that it is the column part that is overestimated, the miscalculation also affects the calculation of the column's resistance to out-of-plane transverse shear, leading to an exponential change of h^3. If the calculated column is long, as in most projects, such an error will be negligible. However, if the calculated beam-element is extremely short, the overestimated length part will constitute a significant proportion of the whole, rendering the error unacceptable.

2.2 Computational modelling assumptions for beam-elements of RCVSP

Currently, the solution to the beam-elements error problem mainly revolves around avoiding the generation of overlapping parts as the primary approach. For example, one of the main methods adopted is modeling the nodes using solid-elements and then coupling them with beam-elements. As a matter of fact, the current software and hardware equipment develop rapidly, and the most accurate way to analyze the force of the entire structure is by directly using solid-elements. However, if full solid-element modeling is used, it is feasible for important buildings. For most civil buildings, the calculations will consume excessive manpower and material resources, leading to increased costs (Schenk et al., 2001; Gould and Scott, 2004). The substructure method based on solid-elements can reduce the calculation scale of the structure by combining multiple units into one superelement. When solid-element nodes are combined with beam-elements, the computational degrees of freedom can be more effectively reduced. However, using the substructure modeling method is more challenging as it involves establishing a comprehensive substructure model and then consolidating the degrees of freedom. Additionally, nonlinear analysis of the building demands a deeper understanding of mechanics, which is often beyond the capabilities of the standard software utilized by most design firms.

VSP has a significant number of upper and lower rib beams and shear keys, making the structure complex. If solid-elements are used for modeling or the substructure method for analysis, it becomes more challenging, ultimately hindering the usability and promotion of this structure. On the contrary, using beam-elements for modeling and calculations is quite hassle-free. The finite element analysis results of the structure can be obtained quickly with the inclusion of plate and shell units on the plate surface. However, the number of intersections between ribs and shear keys in VSP is significant, and the shape of shear keys is predominantly inclined towards a 1:1:1 square. Therefore, the excessive calculation of intersections should not be overlooked. Therefore, the modification of the calculation model of the beam-elements is particularly important in the analysis of VSP.

VSP is derived from the wood structure of the off-seam key structure. This means that the shear key part not only bears the normal load but also fixes the upper and lower rib parts of the structure. Therefore, the shear key part often requires higher stiffness. For example, it is common to use stiffening plates to reinforce the shear keys in steel VSP (Figure 2.b). Shear keys in RCVSP exhibit good compressive properties and are reinforced to enhance their ability to resist shear forces. Usually, the reinforcement is placed at the intersection of the key-beam, where the anchorage at the end of the reinforcement is intricately bundled. As a result, the actual strength of the intersection differs from the rest of the section, making it challenging to calculate the top and bottom ribbed beams while considering the intersection as a continuous material strength.

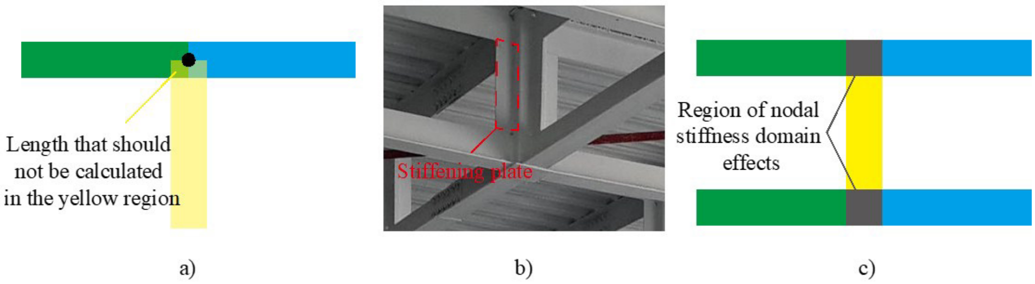

Figure 2. *Node stiffness domain effect*

To perform a more accurate simulation of RCVSP, it is necessary to consider the effect of the self-weight at the end of beam-elements and the effect of the key-beam intersection section. Taking into account the above-mentioned structural characteristics of RCVSP, the assumption of the existence of nodal stiffness domain effects at the key-beam interactions of this structure is proposed (see Figure 2.c): the stiffness of the intersecting part differs from the rest of the calculated part. It is necessary to re-specify the calculation length of the actual beam-elements, as it cannot be calculated directly at the modeled node. To edit the calculation of this part, the following steps must be carried out.

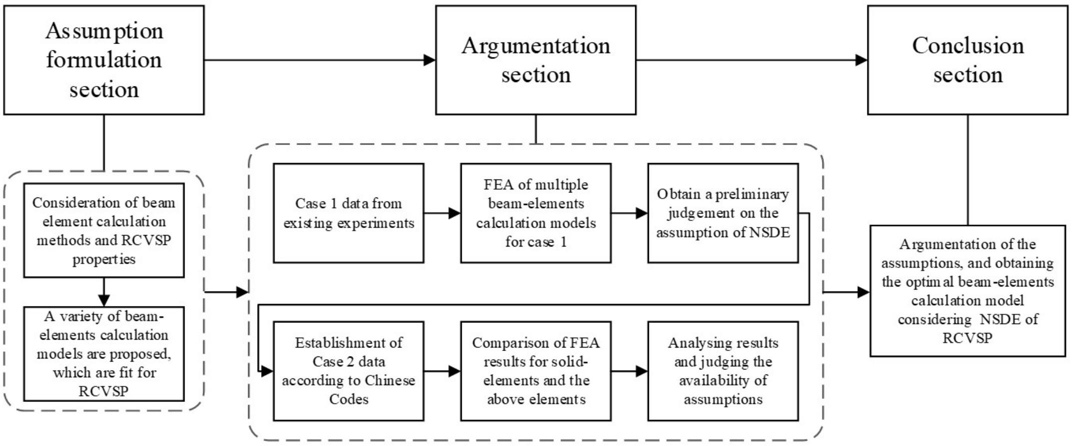

Figure 3. *Flowchart of argumentation*

As shown in Figure 3., FEA of RCVSP is carried out by setting up beam-elements with different calculation methods according to the assumptions, and the differences in the calculation methods are mainly reflected in the performance of NSDE of beam-elements. By comparing the existing experimental data and comparing the results with the solid-elements analysis, the most suitable beam-elements calculation model for RCVSP is finally obtained.

3 Comparative analysis of experimental case

This case was a scaled-down model experiment of a three-story reinforced concrete frame shear wall VSP structure in the Key Laboratory of Structural Engineering of Guizhou Province in 2000(2000).

3.1 Model data

The whole structure was designed according to the actual project and then constructed by using 1:3 scale. The left side of Figure 4. shows the plan view of the scaled-down experimental model, with the panels arranged in prefabricated RCVSP, and the points identified in Figure 4. are the data measurement points (MP-i) on the third panels of the experimental model. The right side of Figure 4. shows a cross-section of the model, with the shape data of VSP shown at the bottom. The model used a single panel of the original design in the ground floor only, with additional panels at the lower rib beams in the second and third floors to increase the overall stiffness of the model.

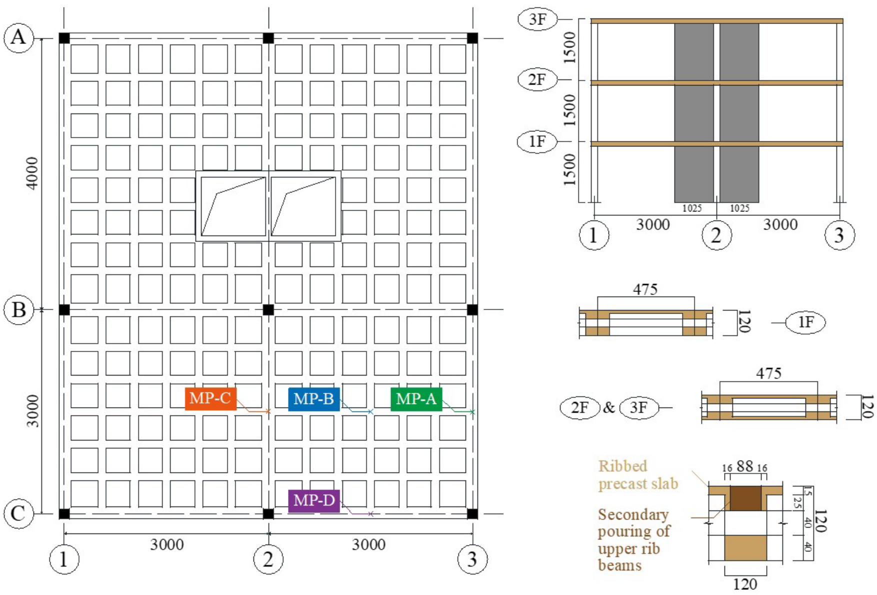

Figure 4. *Experimental model data*

The model was processed by assembly construction technology, the lower rib beams and the shear key part were cast-in-situ once, and the upper rib beams were cast-in-situ secondary after the precast concrete panels were installed. The fabrication materials of the experimental model and their properties are shown in Table 1. below.

Table 1 Mechanical properties of experimental structural materials

Categories	Names	Cement mark	Cubic mean strength	E_c
Concrete	Cast-in-situ C30	525#	33.2MPa	3.08 x 10^1GPa
	Secondary cast-in-situ C35	525#	37.4MPa	3.19 x 10^1GPa
Rebar	Names	Caliber	Yield strength	*Ex*
	Rebar	8mm	320 MPa	1.94 x 10^2GPa
		6.5mm	340 MPa	2.02 x 10^2GPa
	Disc steel wire	4mm	353 MPa	2.15 x 10^2GPa
		2.2mm	332 MPa	2.09 x 10^2GPa

Considering the convenience of structural design and analysis and the applicability of the analysis results, the commercial architectural design software MidasGen and the commercial finite element software Ansys were used to perform linear and nonlinear analyses of the computational models, respectively.

3.2 FEA of linear part

FEA of linear part does not consider the reinforcement and concrete yield damage, and only the modulus of elasticity data and geometric data of the materials in the table are used for modelling. The beam unit and slab unit within the software are utilized for modeling. The upper and lower ribbed beams have cross-sections of 120mm x 40mm, 150mm x 40mm, and 200mm x 40mm. The 120 mm section beam-elements simulates the internal upper and lower rib beams. The 150 mm section beam-elements simulates the upper and lower rib beams in the outermost circle, and the 200 mm section beam-elements simulates the two upper and lower rib beams in the middle cross part. The cross-sectional size of the shear keys varies with the size of the intersecting portion of the beam. The cross-section of the column is 150 mm x 150 mm. The slab is 15mm thick, and the shear wall is 80mm thick. For NSDE of RCVSP, five types of beam-elements calculation models were designed using the function of beam end stiffness in MidasGen (Figure 5.). FeaR1 is the ideal beam-elements modeling calculation model that fully considers NSDE; FeaR2 to FeaR4 are the beam-elements modeling calculation models that do not fully consider NSDE, which are mainly compared with the calculation model FeaR1; FeaR5 is the conventional beam-elements modelling computational model without considering NSDE.

The nodal displacement data of the five measurement points and the actual model data are taken out to obtain Figure 6. The FEA results in Figure 6. correspond to the values in parentheses on the upper side, and the experimental model results correspond to the values on the lower side, and the value of the increase of the data on the upper side is the same as that of the experimental article. For the error shown in the figure, it is hypothesized that it is caused by the following two points: 1) Due to the need to consider the structural resistance to punching and shearing in the experiment, the upper and lower ribs and beams of RCVSP near the plate-column nodes are constructed so that the hollow-bellied structure of the upper and lower ribs of the plate-column nodes is changed to a solid-bellied beam structure, which improves the overall stiffness of the experimental model. At this point, in order to obtain more convincing data, the finite element modeling of this structure was carried out exactly as expressed in the drawings. 2) when the experimental model was subjected to static loading, the displacement was almost zero when the force was at the stage of 2.25kN/m^2, which is believed to be a problem of the measurement accuracy of the experimental displacement gauge at that time. The above two points are one of the main reasons for the error between the calculation results of the finite element model and the measurement results of the actual model.

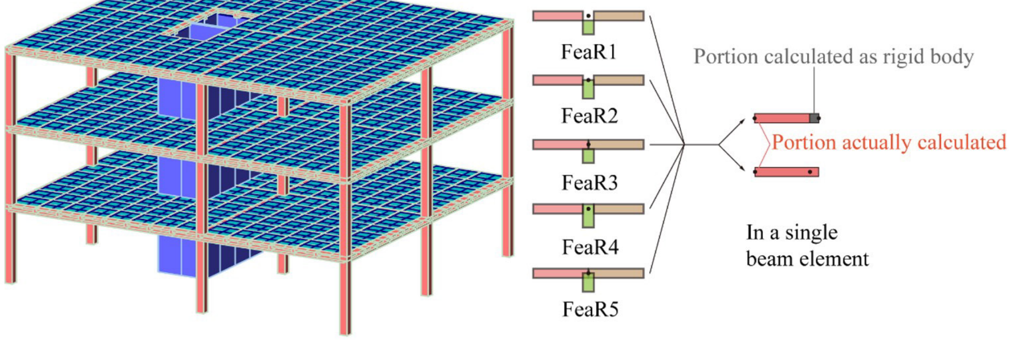

Figure 5. *Five computational models under MidasGen*

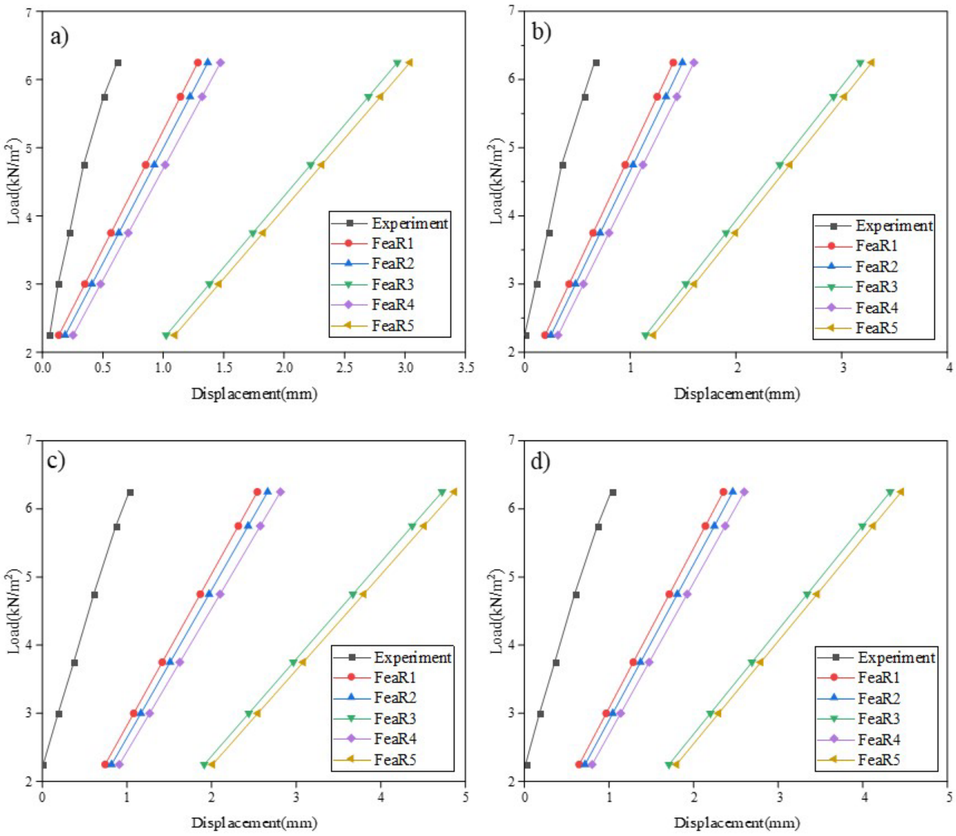

Figure 6. *Load-displacement curve. a)Load-Displacement Curve at Measurement Point A; b)Load-Displacement Curve at Measurement Point B; c)Load-Displacement Curve at Measurement Point C; d)Load-Displacement Curve at Measurement Point D.*

Comparison of the experimental data with the finite element calculation results shows that the overall stiffness of the FeaR1 model, which fully considers NSDE, is the largest, and the calculated data are closer to the experimental data than the other models; and the conventional beam-elements modeling is not enough to support the calculation needs of linear part, and there is a large gap between the calculated results and the model stiffness curves and the experimental data; When NSDE is considered to different degrees, the stiffness of the calculated model is improved to different degrees: Among them, the stiffness of the upper and lower rib beams has a larger effect on the overall model stiffness; the nodal stiffness of the shear key has a smaller effect on the overall model stiffness. Taking the 6.25kN/m^2 data of each measurement point and calling it the maximum deformation D_{FeaRi} of the model, the stiffness restoration rate function η_i is set as:

$$\eta_i = 1 - \frac{D_{FeaRi} - D_{Exp.}}{D_{FeaR5} - D_{Exp.}} (i = 1, 2, 3, 4) \tag{4}$$

Calculations yielded the following Table 2.

Table 2. Stiffness restoration rate at the measurement point

	η_1(%)	η_2(%)	η_3(%)	η_4(%)
A	72.42	68.98	4.32	64.68
B	60.55	57.37	3.78	53.46
C	61.26	58.04	3.81	54.12
D	71.60	68.29	4.10	64.14
Average	66.45	63.17	4.00	59.01

The best performance in terms of stiffness restoration rate is achieved by the calculation model with full consideration of NSDE, which can effectively recover about 66% of the stiffness loss caused by the conventional beam-elements calculation model; the other calculation models with incomplete consideration of NSDE also perform well, but they are not as good as the FeaR1 calculation model. From this, we know that the beam-elements calculation model of RCVSP in the linear stage has obvious NSDE, and there is a large gap between the conventional beam-elements calculation model and the experimental model data; the beam-elements calculation model that fully considers NSDE can effectively shorten this part of the error.

3.3 FEA of nonlinear part

The experimental model was loaded with a damage load of 0-15.25kN/m^2 to obtain the measured point displacement data of the experimental model under different loads. In order to compare the experimental data, the commercial FEA software Ansys software, which is more specialized, was used for the calculation.

For building FEA model, Beam188, Shell63, and Reinf264 elements of the software were used for modeling. The upper and lower ribbed beams, shear keys, and columns in the structure are modeled using Beam188 elements, while the reinforcement inside the members is simulated using Reinf264 elements. The dimensions of the beams and the location of the reinforcement are illustrated in the Figure 7. and Table 3. below. The dimensions of the column are 150mm x 150mm with 6.5mm caliber reinforcement throughout. The shear keys are similar to the columns, but at the intersection of beams of different sections, there are shear keys with cross-sections of 120mm x 200mm, 150mm x 200mm, 120mm x 150mm et al. All of them have 6.5mm caliber reinforcement internally, and their distribution is illustrated in the Figure 8. and Table 4. below. The Shell63 element is utilized to simulate the plate and the shearwalls. No reinforcement is incorporated within the plate and the shearwalls; only the self-weight of the plate and the shearwalls is taken into account, and their damage is not considered. The entire structure is modeled as Figure 9. shown below.

Table 3. Rebar data in rib beam

rib beam	position	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*a*	*b*	*c*	*d*
120 x 40	upper	2.2	2.2	2.2	2.2	2.2	2.2	-	-	5	10	-	-
	lower	2.2	2.2	2.2	4	4	4	8	8	5	5	7	32.5
150 x 40	upper	2.2	2.2	2.2	2.2	2.2	2.2	-	-	5	10	-	-
	lower	2.2	2.2	2.2	4	4	4	8	8	5	5	7	40
200 x 40	upper	2.2	2.2	2.2	2.2	2.2	2.2	-	-	5	10	-	-
	lower	2.2	2.2	2.2	4	4	4	8	8	5	5	7	52.5

Note:All values are in millimeters(mm).

Table 4. Rebar data in *columns and shear keys*

cross-section	*a*	*b*
150mm x 150mm	5mm	5mm
120mm x 120mm	10mm	10mm
120mm x 150mm	10mm	10mm
120mm x 200mm	10mm	10mm
150mm x 200mm	10mm	10mm

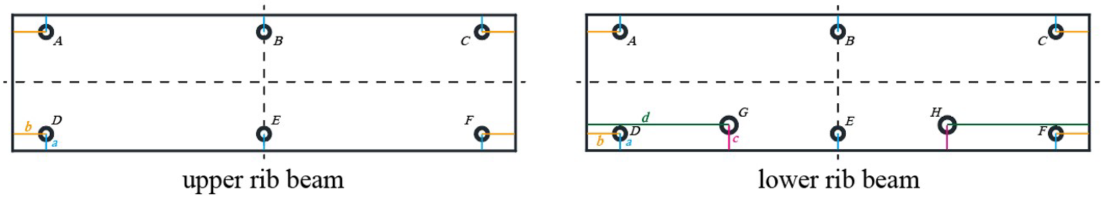

Figure 7. *Rebars position diagram of upper and lower rib beams*

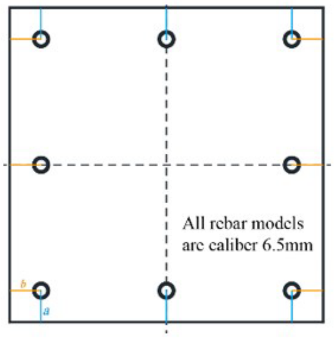

Figure 8. *Rebars position diagram of columns and shear keys* **[[Q2: Q2]]**

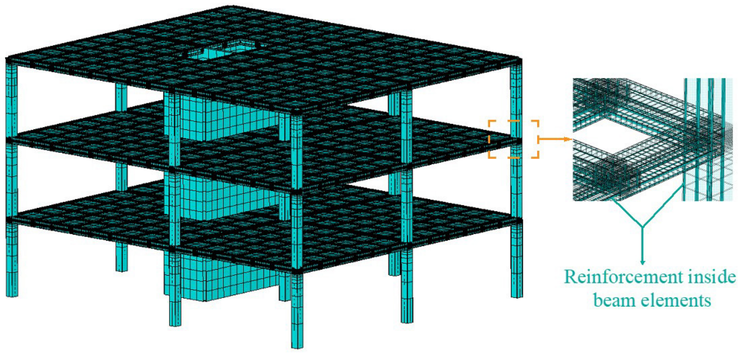

Figure 9. *Modelling by Ansys*

The concrete and rebar principal structure of the calculation model uses the uniaxial force curves(Figure 10) specified in the GB50010-2010 (Chinese Standard, 2015) Code for the Design of Concrete Structures (Chinese Standard, 2015) and the data in the Table 5. NSDE are considered in Ansys by modeling the original beam-elements with multiple segments, adding computational nodes to the ends of the original beam-elements and increasing the stiffness of the end beam segments. In the treatment of the modeled rigid body, the modulus of elasticity is given more than 10^5 times of the original so that the rigid body part has a significant strength difference from other parts, and the rigid body displacement of the end beam section is increased. Considering the results of the upper linear part, finite element calculations are performed only for FeaR1, which fully considers NSDE, and FeaR5, which is modeled as a conventional beam-elements, and compared with the experimental data (Figure 11).

Table 5. Concrete Simulation Parameters

Concrete	C30	C35	Rebar	2.2mm	4mm	6.5mm	8mm
$f_{t,r}$	3MPa	3.5MPa	E	2.09×10^2GPa	2.15×10^2GPa	2.02×10^2GPa	1.94×10^2GPa
$f_{c,r}$	33.2MPa	37.4MPa	k	2.09x10GPa	2.15x10GPa	2.02x10GPa	1.94x10GPa
$\varepsilon_{t,r}$	118E-6	128E-6	$f_{y,r}$	332 MPa	353 MPa	340 MPa	320 MPa
$\varepsilon_{c,r}$	1640E-6	1720E-6	ε_y	1.59×10^{-3}	1.64×10^{-3}	1.68×10^{-3}	1.65×10^{-3}
$\varepsilon_{cu}/\varepsilon_{c,r}$	2.3	2.1					

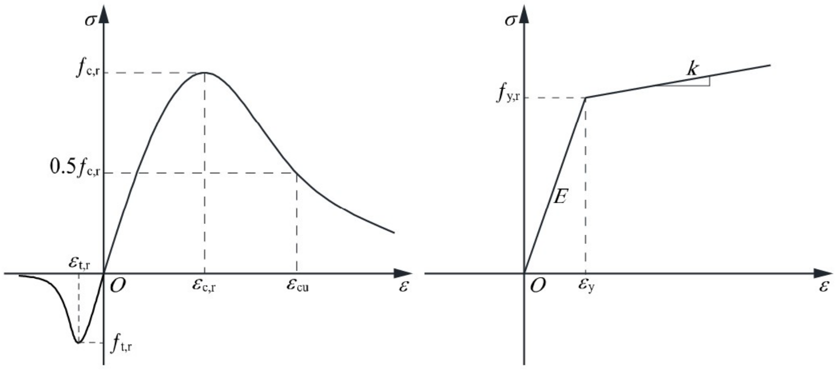

Figure 10. *Uniaxial stress-strain curve of concrete and rebar*

When comparing the experimental data, the results of modeling the beam-elements considering NSDE are significantly closer to the real curves than the results of modeling directly using beam-elements. The overall stiffness of the experimental model is on the high side compared to the designed model, which suggests that the curve simulated by FeaR1 may be closer to the design. In Figure 9.b, the stress-strain curve at point C shows signs of re-stiffening at 10 kN/m^2, which is somewhat different from the finite element results, and it is assumed that this is due to the lack of uniformity of the load applied by the human factor when applying the cast iron pile, which ultimately leads to the irregularity of the deformation of the model at each measurement point.

The computational model of beam-elements with NSDE fully considered enters the yielding stage similar to the real experimental results, and almost enters the yielding stage under the same load; the computational results of the modeling using only beam-elements enters the yielding stage earlier, and the overall stiffness is far less than that of the experimental model, which cannot show the characteristics of RCVSP with good stiffness. The reasons for this phenomenon are analyzed and summarized as the following two points: 1) the beam-elements calculation error caused by the excessive number of nodes of VSP has a large impact on the overall stiffness of the structure, and the gap is relatively significant. There is also a significant difference when comparing the beam-elements calculation model that fully considers NSDE; 2) considering that the experimental data have errors and the data points are not as continuous as those of the finite element calculation results, which further enlarges the gap. From this, it can be concluded that when beam-elements are used to analyze the RCVSP under nonlinear analysis, beam-elements calculation model that fully considers NDSE is closer to reality, while the conventional beam unit is not very useful as a data index for reference.

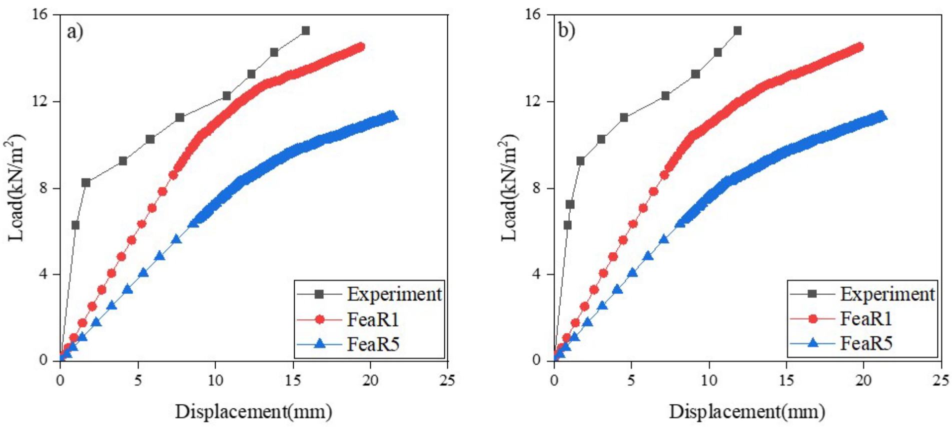

Figure 11 *Load-displacement curve. a)Load-Displacement Curve at Measurement Point B; b)Load-Displacement Curve at Measurement Point C.*

Comprehensive linear part and nonlinear part, can be obtained: RCVSP beam- plate elements FEA, NSDE exists, and can have a greater impact on the stiffness of the structure. FEA of RCVSP using conventional beam-elements cannot meet the requirement of its calculation accuracy; FEA of RCVSP using beam-elements that fully consider NSDE can recover most of the stiffness errors brought by the conventional beam-elements, and its calculation accuracy is greatly increased on the basis of the conventional beam-elements.

4 Comparative analysis of solid-elements

In the previous chapter, the existing experimental data were used as a benchmark to compare the degree of fit of beam-elements model results of different modeling methods, which proved that there is a NSDE in FEA of the beam-plate elements of RCVSP, and that the beam-elements model that fully considers NDSE is more suitable for the RCVSP than the direct use of the conventional beam-elements calculation model. However, due to the error of the results caused by the experiments are too old, it is difficult to get the error of this modeling method with the real data and the advantageous range of direct beam-elements modeling.

4.1 Example model data

According to the Technical Specification for Structures of RCVSP (China, 2005, 2014), Design of a corner supported RCVSP. The plane is 16m × 16m, the grid number is 8 × 8 and the thickness are 0.7m. The upper and lower ribbed beams have a cross-section height of 0.2m and a width of 0.4m. The face laminate is 0.1m thick (Figure 12). The RCVSP was modeled using Solid45 elements, Beam188 elements, and Shell63 elements in Ansys software, with a unit length of solid-elements is 50 mm. The rib beam and shear key portions of this RCVSP is modeled separately using three modeling methods: solid-elements, conventional beam-elements, and beam-elements that fully consider NSDE, and the plates is modeled uniformly using Shell63 elements (solid-elements calculation model still uses Solid45 elements).

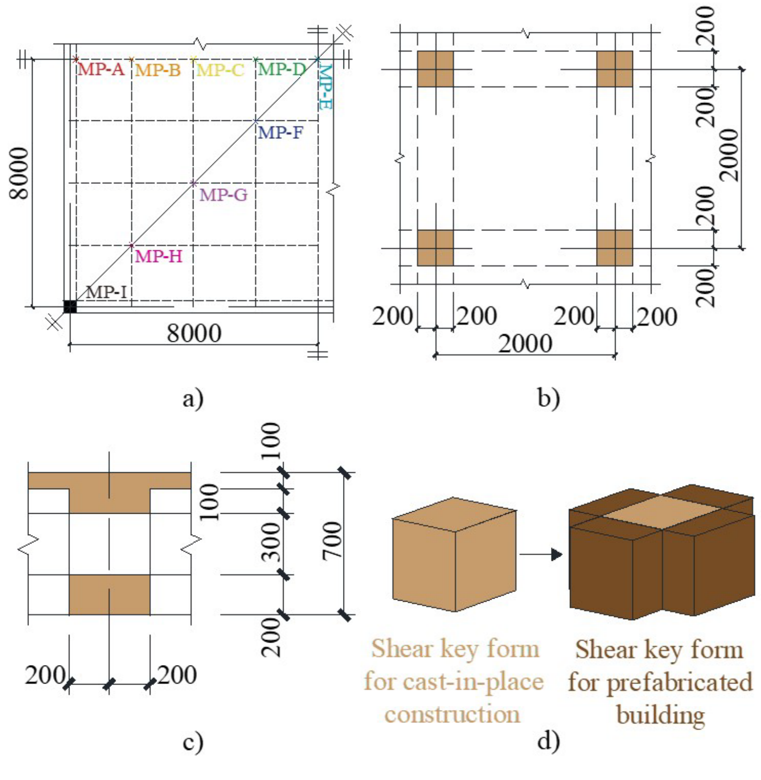

Figure 12. *Data for the algorithm. a)Measurement point locations and size of VSP; b)Individual grid sizes for VSP;* c)Data for upper and lower ribbed beams and slabs; d)Shape change of shear key.

Concrete strength class C30.(E_c=30GPa) With the latest research and engineering practice of assembled RCVSP, the original shear key portion was changed from rectangular to cruciform. That is, a portion of the length was added toward the rib to increase overall stiffness (Figure 12d and 13). In this case, the extension length is calculated to be half the length of the original shear keys.

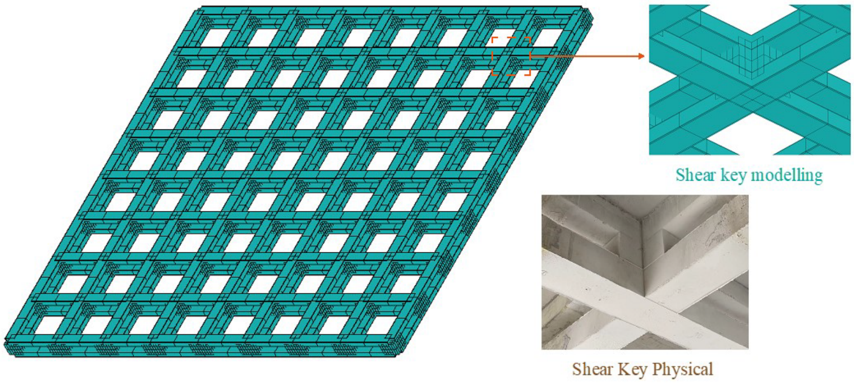

Figure 13. *Shear key modelled by Ansys*

4.2 FEA for uniform loads

Considering the self-weight of the structure, the same load was applied to the three computational models and the nodal displacement data were obtained to obtain Figure 14. below.

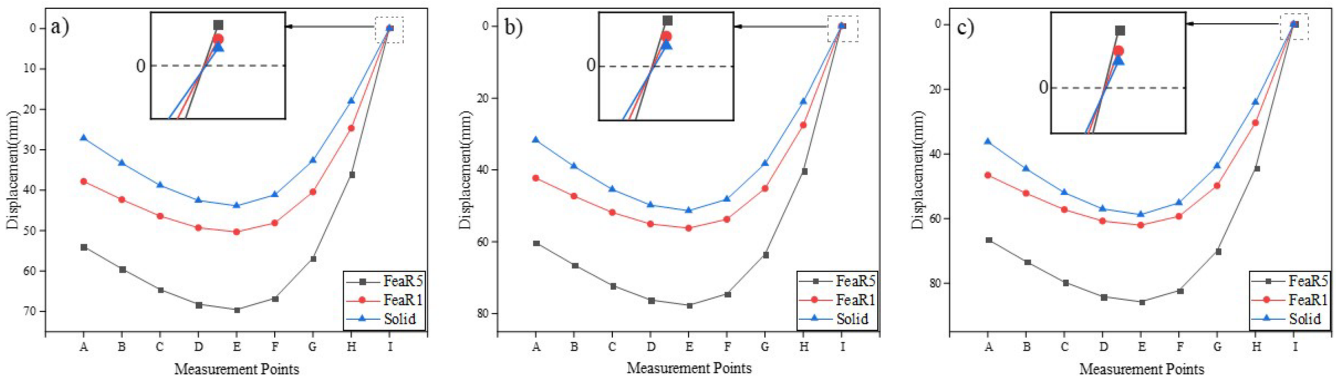

Figure 14. *Nodal displacements under static loads. a)6kN/m^2; b)8kN/m^2; c)10kN/m^2.*

Figure 14. shows that the displacement data of beam-elements calculation model with full consideration of NSDE is very close to the data of solid-elements calculation model. When the load is increased, the data of beam-elements calculation model with full consideration of NSDE is closer to the data of solid-elements calculation model. There is no NSDE in the solid-elements calculation model, and its stiffness best represents the actual stiffness of RCVSP, which is the highest among the three calculation models. The conventional beam-elements calculation model exhibits a significant difference in stiffness compared to the solid-elements model because it lacks NSDE. However, once NSDE is taken into account, the stiffness significantly improves, aligning with the test results presented in the previous section. Taking the data of measurement point E to calculate the Stiffness restoration rate, the values of η_1 under load increase are obtained as follows: 74.88%, 81.56% and 87.92%. That is, η_1 shows a positive correlation with load increase. It indicates that beam-elements calculation model, which fully considers NSDE under the said high load, can be more equivalent to compensate the error brought by conventional beam-elements.

Taking the data of some measurement points in the span of the panel of the two beam-elements calculation models, the average error is calculated using the data of solid-elements calculation model as a reference, and the following Table 6. is obtained.

Table 6. Errors in mid-span data for two beam-elements calculation models

Load(kN/m^2)	FeaR1(%)	FeaR5(%)
6	15.82	60.36
8	10.51	52.98
10	6.56	47.48

Table 6. shows that the modeling calculation with the conventional beam-elements has a large error compared with solid-elements calculation results, although with the growth of the load is slightly reduced, but the results still do not

have the value of the reference; fully consider NSDE of beam-elements calculation model and solid-elements calculation results of the trend is consistent, and the error is smaller at the same time, in the larger load can be more close to the results of solid-elements. Taking the load of 10 kN/m^2 as an example, the beam-elements calculation model with NSDE fully considered can recover 87.92% of the conventional beam-elements stiffness, which is more valuable as a reference value for practical engineering. Moreover, the conventional beam-elements calculation model has obvious warping phenomenon at the four corners of the beam, and the beam-elements calculation model with full consideration of NSDE can effectively reduce the material loss and design difficulty, which increases the structural stiffness and reduces the warping of the four corners. It can be obtained that when solid-elements calculation model is used as a reference, the conventional beam-elements calculation model for RCVSP has no reference value, and the beam-elements calculation model that fully considers NSDE can obtain smaller error calculation results.

4.3 FEA of seismic response

The seismic response of the structure is also one of the practical considerations in engineering. The first 15 seconds of north-south acceleration data of El-centro seismic wave was selected to load the three computational models in time, and the computational damping was taken as 0.05, and the acceleration was taken as 9.8. The deformation data of the three models with time were obtained (Figure 15.).

Figure 15. shows that the overall trend of the seismic response of the three computational models is the same, but there are more obvious differences in the extreme point data. The beam-elements model, which fully considers NSDE, is closer to the solid-elements calculation results, compared with the conventional beam-elements model. A large number of errors are reduced, and several exaggerated deformations are recovered. Conventional beam-elements model in the overall looser, in a number of turning point displacement error greatly exceeds the solid-elements results of 50%, increasing the design of the structure needs to be safe redundancy. In the calculation time of the model, the solid-elements modal is used for a long time, about 40(min), and the beam-elements model with NSDE is used to save the calculation time, about 12(min), which can be used to calculate the seismic response of the RCVSP. In Figure 16., it is shown that the beam-elements calculation model, which fully considers NSDE, increases the computation time compared to the conventional beam-elements calculation model. Considering that the new beam-elements increases the amount of computation at the end nodes, such a difference in computation time is within reasonable limits. After further optimization by the algorithm, the gap will be further reduced. The direct use of conventional beam-elements for modeling will result in large errors in the data, which will lead to an increase in the cost of structural construction.

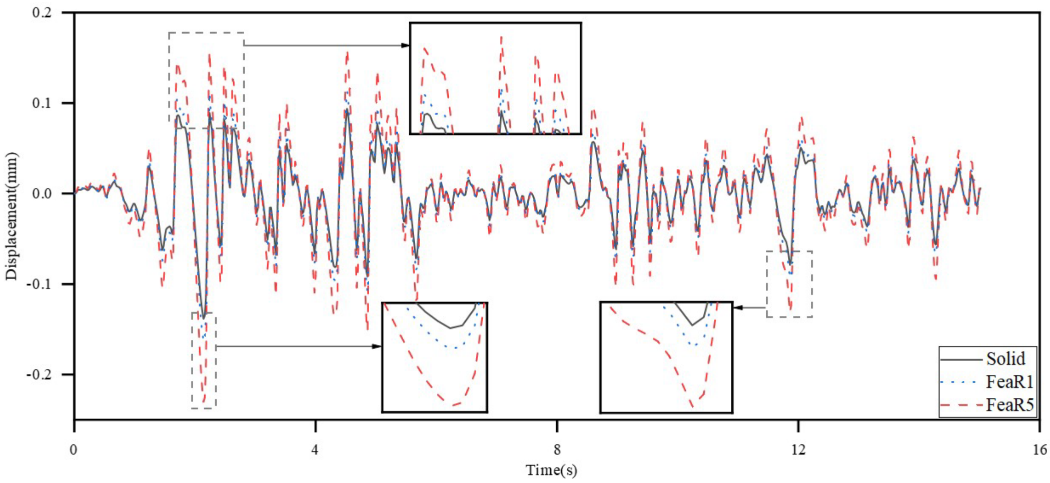

Figure 15. *Comparison of time-phase analyses*

Comprehensive comparison of the static dynamic analysis part, it can be concluded that the use of conventional beam-elements for FEA of the RCVSP compared with the solid-elements analysis results of the error is large, cannot be used as a reference for engineering design; The use of the beam-elements fully consider NSDE of FEA of the RCVSP and the solid-elements analysis results are similar to the error of the extreme point within 16%, and the reduction in the size of the computational scale brought about by the calculation of efficiency is greatly increased, and only a plate can reduce the time of about 3 times.

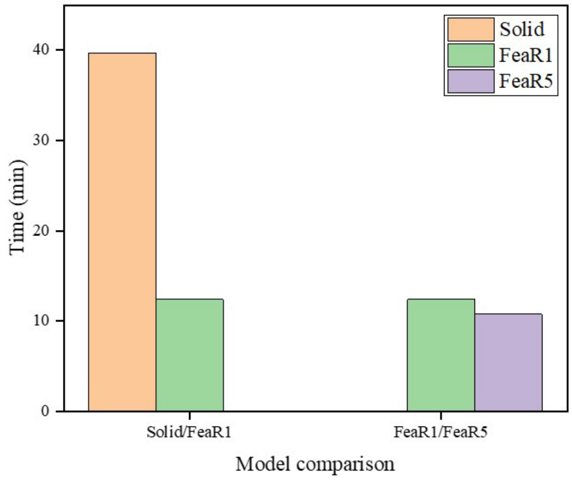

Figure 16. *Calculation time*

Comparing the three calculation models, it can be concluded that the advantage of the beam-elements calculation model that fully considers NSDE over the conventional beam-elements calculation model lies in the increased calculation accuracy. The advantage of the beam-elements calculation model that fully considers NSDE over the solid-elements calculation model lies in the fact that the computational efficiency tends to be closer to that of the conventional beam-elements calculation model, and the computational time is significantly reduced.

5 CONCLUSION

After comparing the results of different kinds of calculation models for RCVSP and comparing and analyzing the existing experimental data, the following conclusions can be drawn:

1. In the beam-plate elements calculation model of RCVSP, there is a NSDE in the FEA using the beam-elements, and this effect will bring a large error to the calculation results.
2. The calculation results of finite element modeling of RCVSP using conventional beam-elements do not match the actual results, and the calculation model using conventional beam-elements does not meet the calculation accuracy requirements.
3. The finite element modeling of RCVSP using beam-elements that fully consider NSDE gives results that are similar to the real results and can recover the stiffness loss caused by conventional beam-elements. The stiffness recovery is about 66% compared with the experimental data and about 80% compared with solid-elements data.
4. The use of fully consider NSDE of the beam-elements for nonlinear analysis of the RCVSP can have the same yield stage load with the real results, closer to the RCVSP load displacement change rule.
5. The use of the beam-elements that fully consider NSDE for the engineering design of RCVSP can effectively reduce the demand for stiffness caused by the direct use of conventional beam-elements calculation errors, thus reducing resource consumption.
6. The finite element modeling of assembled RCVSP using the beam-elements that fully consider NSDE still gives an error of not more than 16% when compared with the solid-elements, which is a good performance in the simulation of assembled shear keys.
7. Comparing the three calculation models, the advantage of the beam-elements that fully considers NSDE is evident. It offers calculation efficiency similar to that of the conventional beam-elements calculation model and the calculation accuracy of the solid-elements calculation model, making it suitable for practical project applications.

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