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Numerical Investigation of Structural Response of Corrugated Blast Wall Depending on Blast Load Pulse Shapes

Abstract

Hydrocarbon explosions are one of most hazardous events for workers on offshore platforms. To protect structures against explosion loads, corrugated blast walls are typically installed. However, the profiles of real explosion loads are quite different depending on the congestion and confinement of Topside structures. As the level of congestion and confinement increases, the explosion load increases by up to 8 bar, and the rising time of the load decreases. This study primarily aims to investigate the structural behavior characteristics of corrugated blast walls under different types of explosion loadings. Four loading shapes were applied in the structural response analysis, which utilized a dynamic nonlinear finite element method.

Keywords

Explosion loads, loading pulse shape, nonlinear finite method, structural characteristics

Jung Min Sohn ^a Sang Jin Kim ^b

^a Department of Naval Architecture and Marine Systems Engineering, Pukyong National University, 45, Yongso-ro, Nam-gu, Busan, 48513 Republic of Korea
^b The Korea Ship and Offshore Research Institute (The Lloyd's Register Foundation Research Centre of Excellence), Pusan National University, 2, Busandaehak-ro, 63 beon-gil, Geumjeong-gu, Busan, 46241, Republic of Korea, sangjin@pusan.ac.kr

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1 INTRODUCTION

In the coming years, it is predicted that there will be a significant shift worldwide from searching for hydrocarbons onshore to looking for them in ultra-deep waters. However, there is an inherent risk of hydrocarbon releases associated with the production, processing, storage, and transportation of hydrocarbons. These processes can cause hydrocarbon explosions and fires, which are the most hazardous accidents that can occur on both offshore and onshore installations. In addition, the explosions involve extreme explosive actions, which can cause serious casualties, property losses, and marine pollution (Paik, 2011). After the publication of the Cullen Report (Cullen, 1990) on the Piper Alpha tragedy, the offshore industry conducted a great deal of research directed toward understanding the characteristics of hydrocarbon explosions. Despite these efforts, the Deepwater Horizon accident occurred in the Gulf of Mexico on the 20th of April, 2010. As part of the growing interest in hydrocarbon explosions, safety critical elements have come to encompass any objects or items intended to mitigate the consequences associated with a major blast load or which aim to contain an explosion to minimize the risks of oil storage, transportation, and usage (Boh et al., 2004; API, 2006; Chen, 2011; ISSC, 2015). This study considers blast walls to be one of most frequently used safety systems against explosion loads; they are also the best method in terms of cost-benefit considerations. Blast walls consist of specially designed lightweight panels, which are designed specifically for each building and consider the internal threat to the relevant wall. They are installed to provide a physical separation between hazardous and non-hazardous areas of the topside to avoid detonation toward adjacent areas in case of explosion accidents (Schleyer et al., 2007). They are also installed to protect people and vital equipment from the hazards of gas explosions.

The designer has the difficult task of predicting what type of explosive load a structure may be exposed due to the numerous uncertainties inherent to an explosion. Generally, industry, classification societies, and international/national organizations are used to idealize random explosion loads as symmetric triangular shapes since these best matched an actual explosion loading with respect to such attributes as total impulse, peak pressure, rise time, and pulse duration (Czujko, 2001; Oil & Gas, 2007; DNVGL, 2010; Paik, 2011; ABS, 2013; LR, 2013). However, explosion loads can have quite different patterns depending on the levels of confinement and congestion (Catlin et al., 1993; Bjerketvedt et al., 1997; Selby and Burgan, 1998; Kim et al., 2014; Paik et al. 2014). In addition, the loading shape is expected to have a profound influence on the dynamic structural response because the loading shape is directly linked to the explosion loads including impulse and peak pressure (Li and Meng, 2002; Florek and Benaroya, 2005; Chernin et al., 2016).

The aim of this study is to investigate the structural response characteristics of blast walls under different explosion loading shapes, which represent several different phenomena depending on the layout of structures; in other words, the effects of congestion and confinement are investigated. For this purpose, a series of numerical computations are performed to investigate the effect of the type of blast loading shape on the dynamic structural response depending on the following domains and conditions: elastic and elasto-plastic and quasi-static, dynamic, and impulse. Particular attention was given to the ductility level analysis of blast walls.

2 CORRUGATED BLAST WALL

2.1 Geometric Characteristics

Typical blast walls are thin, corrugated stainless steel panels or assemblies of stiffened steel plates supported by frames (Czujko, 2001; Schleyer et al., 2007; Paik, 2011). Here, 4m high stainless steel panels were corrugated from top to bottom are adopted. They were welded top and bottom and down both sides to the primary steel work through angle connections. Schematic drawings are shown in Figure 1 (HSE, 2003).



(a) A cross section of the corrugated panel with loading direction (b) Details of connection

Figure 1: Main dimensions of blast wall in the present study (HSE, 2003).

2.2 Material Characteristics

Structural behaviors under explosion loadings should consider the effects of the strain rate on the yield stress. The material model used in ANSYS/LS-DYNA (2015) is the "Material type 24 piece-wise_linear_plastic" model. This is an elasto-plastic material model, and the strain rate is accounted for by using the Cowper and Symonds (1957) model. Dynamic yield stress scales the static yield stress with factors and, which were calculated according to Eq. 1.

$$\frac{\sigma_{\rm Yd}}{\sigma_{\rm Y}} = 1.0 + \left(\frac{\dot{\epsilon}}{\rm C}\right)^{1/q} \tag{1}$$

where, σ_{y_d} and σ_y =yield stress under dynamic and static loads, $\dot{\epsilon}$ =strain rate, and C and q=Cowper-Symonds coefficients.

HSE (2003) conducted dynamic tensile tests of 1/4-scaled blast wall to produce dynamic material characterization data of stainless steel panels, connections, and primary steelwork. The material properties are presented in Table 1, which summarizes the dynamic characteristics of the stainless steel used for the blast panel and connection as well as the mild steel used for the I-beam.

Part	t (mm)	E (GPa)	$\sigma_{\rm Y}$ (MPa)	C (1/s)	q
Corrugated Plate	2	200.0	293.7	1522	5.13
Angle	4	200.0	283.3	2720	5.78
Flexible angle	3	200.0	276.2	429	4.08
I-beam	12	205.8	235.0	40.4	5

Table 1: Dynamic characterization of materials for blast panel and I-beam (HSE, 2003).

3 FINITE ELEMENT MODEL

Blast wall structures can be represented using a Finite Element (FE) model, which is a means for obtaining approximate numerical solutions. FE analysis is the most refined numerical calculation method currently available and is believed to provide the most accurate solutions (FABIG, 1996; Czujko and Paik, 2015). The simulation automatically determines the local load distribution caused

by inertial acceleration and produces time histories of deflections, stresses, and other factors. In the present study, the FE-based software ANSYS/LS-DYNA (2015) is applicable to simulate and analyze nonlinear structural characteristics by considering large deformations and nonlinear material. Sohn et al. (2013) compared structural behaviors between experiment and numerical model in the elastic and plastic region. It proved the accuracy of numerical results under blast loadings.



Figure 2: Nonlinear finite element model together with the applied boundary conditions.

The main structure includes the corrugated panels (height of 3660mm), angle connections, and primary steelwork to assess the global response of blast walls. A fine mesh density was required to capture the local buckling modes of the structures, while quadrilateral elements with four nodes and six degrees of freedom per node were required to analyze the corrugated panels. Sohn (2015) conducted mesh convergence tests for the same blast model and suggested that the element size of 20×20 (mm) (width × breadth) which did not require excessive computation time.

A fully integrated quadratic 8-node element with nodal rotation was assigned to the connections and steelwork to simulate both bending and large strain deformation behavior taking into account material nonlinearities.Four breadth-wise aligned continuous corrugated panels were considered to minimize boundary effects at the measurement point. Symmetry conditions were applied at the two edges of the blast walls. Additionally, the connections were fully restrained along the bottom face and were modelled to share nodes with the primary steelwork to illustrate the welding at the top and bottom edge.

4 PRESSURE-TIME PROFILES

The blast load profiles of explosions are one of the main factors that need to be considered in a structural analysis. The mechanism of hydrocarbon explosions is very complicated and has a high level of nonlinearity. In addition, the shapes of the explosion loads differ greatly according to the type of explosion, congestion, and structural confinement, as shown in Figure 3 (Kim, 2016). Negative phase duration with negative pressure is small and unquantified for a vapor cloud explosion; this value is generally assumed to be zero.



Figure 3: Explosion load profiles according to the congestion and/or confinement of structures (Kim 2016).



Figure 4: Common pulse loading shapes.

There has been a growing interest in the influence of the pulse shape on the dynamic response of a structure due to the variety of possible blast actions and the difficulty of reproducing the exact blast loading. Over the past thirty years a number of studies have been conducted regarding the sensitivity of simulations on the loading pulse shape (Jones, 1975; Youngdahl, 1987). These covered a wide range of topics concerning the dynamic plastic behavior of various structures including the effect of considering finite displacement.

Latin American Journal of Solids and Structures 14 (2017) 1710-1722

An actual explosion, like most blast loads, has a random pressure-time history. Biggs (1964) developed maximum response charts for an elastic-plastic single-degree-of-freedom system subjected to rectangular, linearly decaying, triangular, and gradually applied loads. Those four types of pulses, as shown in Figure 4, were adopted in the present study to perform the numerical computations of a blast wall's dynamic response.

These load shapes can be expressed by a combination of parameters, such as the rise time until peak overpressure, decaying time after peak overpressure, and/or duration. The triangular and gradually applied load pulses have finite rise times, while the other pulses have instantaneous rises to a peak pressure. Furthermore, triangular and linearly decaying pulses have gradual decay characteristics as opposed to the rectangular and gradually applied loads. Further, it can be clearly seen that the triangular and linearly decaying pulses have the same impulse.

4.1 Peak Pressure

A number of peak pressures in a finite time make up the loading pulse; they are organized to understand the effect of a blast loading shape. A static analysis was performed to assign pushover curves, which can be utilized to define whether the peak pressure is in the elastic or elasto-plastic region. Figure 5 shows the static capacity and the corresponding response of a blast wall subjected to static pressure load at the center of a corrugated blast wall.



Figure 5: Result of static push-over analysis of a blast wall.

The pressure was increased until the wall collapsed. The increase in the load capacity was caused by the development of membrane forces. End supports performed a role to resist axial forces and bending moments. A 0.1% plastic strain was defined as the elastic limit where all the reserve strength of the blast barrier was used up. The resulting values were 0.796bar of peak pressure and 28.68mm of deflection. Additionally, the regions above these values were set as elasto-plastic.

Normally, the maximum overpressure used for blast wall design does not exceed 1.5bar based on ductility level blast. However, several explosion loadings have been recorded up to 8.0bar depending on the confinement in Computational Fluid Dynamics (CFD) and experiments (Catlin et al., 1993;

Bjerketvedt et al., 1997; Selby and Burgan, 1998; Kim et al., 2014; Paik et al., 2014). In addition, the effect of loading pulse structural responses may be affected by the magnitude of the peak pressure.

Therefore, this study considers high peak pressures, which can causes structural damage, such as weld failure, local instability, and extensive inelastic deformations. Finally, the elastic region was assumed to be located between 0.5bar and 0.75bar of peak pressure, while the elasto-plastic region was assumed to lie between explosion loads of 1.0 to 3.5bar. These explosion loads applied to the blast walls in a uniformly-distributed and normal to the surface. These explosion loads applied to the blast walls are uniformly distributed and normal to the surface.

4.2 Duration

In the present study, duration is defined as a continuous time for primary blast loadings, i.e., rebound duration was disregarded. Modal analysis was conducted by extracting the natural period to assign three domains, namely the quasi-static, dynamic, and impulsive domains. According to NORSOK (1999), the structural behavior can be idealized depending on the ratio of the duration of the impact actions to the natural period of the structure as follows, with a natural period (T) and duration (τ) ;

- Quasi-static domain: $3 \le \tau / T$
- Dynamic/impact domain: $0.3 \le \tau / T < 3$
- Impulsive domain: $\tau / T \le 0.3$

The first mode shape is the overall response in the corrugated panel with the natural period of 0.0296s, which is shown in Figure 6. Several lowest mode shapes with their corresponding natural periods are related to the local overall response of the individual corrugated panels, owing to continuous modeling of the corrugated panel. The following mode is the response of each corrugated plate intersection at 0.0122s. The duration times were determined such that all three domains were involved in each simulation based on the natural period time. The duration (τ) was set to various values for the impulsive domain (0.005, 0.0065, and 0.008s), dynamic domain (0.01, 0.02, 0.03, 0.05, and 0.07s), and quasi-static domain (0.09 and 0.10s).



Figure 6: Resultant displacements for the modal analysis at 33.752Hz.

5 RESULTS AND DISCUSSION

To account for geometric and material nonlinearities, a nonlinear finite element method was used to assess the effect of the blast loading shape on the structural response characteristics of blast walls exposed to blast events. A time history analysis was performed to trace the entire explosion event from its initiation through to its termination. The overall response of the blast walls subjected to a blast loading was usually dominated by elastic-plastic deformation. Blast walls were generally allowed to respond plastically, allowing them to have greater reserve capacities and ductility. This is because blast walls provide a better means of retaining their integrity under the kind of high loading levels that can cause permanent deformation (HSE, 2000).

In the present study, the ductility ratio was used as a way of distinguishing the structural response based on the assumption that the curvature in the maximum momentum regions increases proportionally with deflection. The ductility ratio is defined as follows;

$$\mu = \frac{\delta}{y_{el}} \tag{2}$$

where, δ =maximum deflection, and y_{el} =deflection at the elastic limit.

Figure 7 shows a comparison of the ductility ratio as a function of duration with varying levels of the peak pressure for four types of loading pulses. The ductility ratio increases as higher pressures are applied. In addition, the ductility ratio linearly increases with increasing duration times until the ductility ratio reaches a constant value. Once the ductility ratio is equal to one, it means that the structural response is elastic (which occurs at a peak pressure of 0.5bar).

An elastic response is dominant when the triangular and gradually applied loading are applied; however, rectangular and decaying loads result in a permanent set at a peak pressure of 0.75bar. We propose that this is because of the instantaneous rising time of the loading pulse affects the structural response in the elastic region. Additionally, the plastic response is shown regardless of the loading pulse type for peak pressures of more than 1.0bar. When the duration is much shorter than the resonant time period, the ductility ratio has a linear relationship with the duration time without depending on the magnitude of the peak pressure in the impulsive domain. In this case, most of the deformation of the structure will occur after the blast loading has diminished.

In addition, a strong transient response can be observed around the resonant time; the deformation of the structure is a function of time, and the response is determined by solving the equation of motion of the structural system. The structural response shows that the ductility ratio steadily tends toward a constant value while the loading is still being applied in the quasi static domain.

The natural period is a distinct point with which the domain can be determined. However, these periods shrink as higher peak pressures and larger loading impulses are applied. This means that the natural period has a reduced effect on the structural response.

The maximum overpressure used for the blast wall design does not normally exceed 1.5bar based on ductility level blast. Therefore, the structural response was closely examined in elastic and elasto-plastic region to understand the effect of the loading pulse shape, as shown in Figure 8.



Figure 7: Ductility ratio as a function of duration time for several types of pressure-time histories with different peak pressures: (a) rectangular, (b) gradually applied, (c) linearly decaying, and (d) triangular.

Two domains were considered: impulsive and dynamic domains. It was found that the ductility ratio of the blast walls under the same uniform peak pressure and duration depended strongly on the pulse shape. The rectangular load induced the largest ductility ratio for the same pressure and duration, followed by the gradually applied load, the linearly decaying pulse load, and finally the triangular load. It is known that permanent deflection can be estimated by considering the relative sizes of the impulses Sohn (2015). A different loading pulse induces a completely different impulse; thus, we concluded that impulse is a critical parameter in determining the structural response of blast walls.



Figure 8: Comparison of the change in ductility ratio for various pulse shapes.

Additionally, the structural response was not proportional to the impulse. A widening gap was observed as higher peak pressures were applied in the dynamic domain. For instance, the impulse of a rectangular load was double that of a triangular load, but its ductility ratio was 4.55 times larger than that of a triangular load at a peak pressure of 1.5bar in the dynamic domain.

Further, linearly decaying and triangular loads had identical impulse values. However, the results show that they behaved differently depending on the domain; they both had similar ductility ratios in the impulsive domain, however, the gap between their responses increased considerably in the dynamic domain. In other words, the rising time is not critical in the impulsive domain, but it is very important in the dynamic domain.

The results in Figure 8 raise the question of how the ductility ratio can change when the impulse remains constant. Figure 9 expresses this question by comparing the four types of loading pulse at the same impulse value. Figure 9(a) demonstrates that the ductility ratios are generally similar regardless of the loading pulse type in the impulsive domain.



Figure 9: The dependency of the ductility ratio with increasing impulse for different loading pulse shapes.



Figure 10: Ductility ratio versus impulse curves for different loading pulse shapes at a specific peak pressure.

Note that duration and peak pressure are not critical but, impulse is important in the impulsive domain. This changes for higher impulses and is related to the differences in their peak pressures. Figure 9(b) shows the results in the dynamic domain. The interesting thing is that the relative strength of the ductility ratio is quite different compared with for constant peak pressure. The largest ductility ratio is observed for a decaying load, followed by triangular, gradually applied load, and finally rectangular load. Herein, the rectangular load shows the lowest peak pressure at the same impulse even though it has the highest impulse at the same peak pressure. The differences in peak pressures lead to the differences of the ductility ratios. The ductility ratio increased more dramatically with changing impulses compared with changing the peak pressure. It can be concluded that the impulse of the blast load is more sensitive to the structural response of blast walls than the peak pressure.

Figure 10 presents the ductility ratio as a function of impulse for a fixed impulse and peak pressure. The largest response was for the rectangular load and the second largest for the decaying load in the elastic region while the second largest was for the gradually applied load in the elasto-plastic region. This means that the rising time is important in determining the structural response in the elastic region. However, the internal energy is greatly increased at higher peak pressures, so the influence of the rising time is reduced. Also, the ductility ratio tends toward a constant value as higher impulses are applied regardless of changes in the duration. It means that the duration time becomes more important as a response to the quasi-static domain. The ductility ratio dramatically increases by changing the impulse rather than changing the peak pressure and duration time. Finally, these results indicate that the structural characteristic of blast walls depends on the amount of applied momentum, rather than peak pressure and duration time.

6 CONCLUSIONS

Different explosion processes produce a variety of pulse pressure histories on the structure. The designer has the difficult task of predicting the effect of an explosive load on a structure owing to the numerous uncertainties inherent to an explosion. This paper examined the effect of the loading

pulse shape on the ductility ratio of a blast wall. In accordance with the objectives of the study, computational techniques for carrying out FE analysis to determine the response of the blast wall were established. Based on the numerical analysis, we came to several conclusions;

- Ductility ratio becomes higher as the higher pressure and longer duration time are applied. Elastic behavior was shown for the most part under 0.75 bar, while plastic behavior is dominant over 0.75 bar. Also, it is known that rising time is critical in elastic region.
- The structural behavior is significantly influenced by the natural period. The effect of natural period is reduced as higher peak pressure and larger loading impulse are applied.
- The effect of the shape of the load time history on the structural characteristic of blast walls were studied using the ductility ratio. Note that duration and peak pressure are not critical but impulse is important in the impulsive domain. The rising time and the level of impulse have profound influence on the structural characteristic of blast walls in dynamic domain.
- The ductility ratio is dramatically increased by changing the impulse rather than the peak pressure and duration time. Finally, these results indicate that the structural characteristic of blast walls depends on the amount of applied momentum, rather than peak pressure and duration time.

The insights gained in the present study will be very useful for the structural design of corrugated blast walls under explosion loads. Future studies are expected to compare the outcomes of using a nonlinear FE analysis and a single-degree-of-freedom method.

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