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Large amplitude free vibration of micro/nano beams based on nonlocal thermal elasticity theory

Abstract

This paper is concerned with the nonlinear free vibration of a heated micro/nano beam modeled after the nonlocal continuum elasticity theory and Euler-Bernoulli beam theory. The governing partial differential equations are derived from the Hamilton variational principle and von Kármán geometric nonlinearity, in which the effects of the nonlocality and ambient temperature are inclusive. These equations are converted into ordinary forms by employing the Kantorovich method. The solutions of nonlinear free vibration are then sought through the use of shooting method in spatial domain. Numerical results show that the proposed treatment provides excellent accuracy and convergence characteristics. The influences of the aspect ratio, nonlocal parameter and temperature rise parameter on the dimensionless radian frequency are carefully investigated. It is concluded that the nonlocal and temperature rise parameters lead to reductions of the nonlinear vibration frequency, while the influence of the nonlocal effect decreases with an increase in the aspect ratio.

Keywords

Euler-Bernoulli beam; nonlocal elasticity theory; nonlinear free vibration; Kantorovich method; shooting method.

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1 INTRODUCTION

Since the initial discovery of carbon nanotubes by Iijima (1991), a new attractive topic has come under ever-increasing research scrutiny for micro/nano-sized structures. The exceptional physical, chemical, mechanical, electronic and thermal properties of this structure have led to a wide range of applications (Poncharal et al., 1999; Guz et al., 2007). Due to the presence of small scale effects which are related to the atoms and molecules that constitute the materials at micro/nano scale, atomic modeling method, such as molecular dynamics simulation (Tuzun et al., 1996) is certainly conceptually valid for the accurate mechanical analysis. However, the approach is computationally exorbitant for micro/nano structures, especially for large sized atomic systems. As experiment in micro/nano scale is difficult to conduct and control, continuum modeling is becoming an alternate to atomistic method.

Conventional continuum models may lead to erroneous results as for small scales continuum assumption may not hold valid. To overcome this drawback, the development of scale-dependent continuum theories, such as nonlocal elastic theory (Eringen, 1983 and 2002), strain gradient elastic theory (Kahrobaiyan et al., 2011; Aifantis, 1999), and couple stress theory (Mindlin and Tiersten, 1962; Yang et al., 2002), is initiated. Among these theories, the theory of nonlocal elasticity has received much popularity in the mechanical analysis of micro/nano structures whilst the results show a good consistence with that in molecular dynamics (Chen, 2004).

In 1970s, Eringen (1972) pioneered the nonlocal elasticity theory, where the classical or local continuum mechanics was modified by specifying the stress at a reference point can be considered to be a function of the strain field at every point in the domain to account for the scale effect in elasticity. In this regard, the internal size or scale could be considered in the constitutive equations simply as a material parameter. Such a nonlocal elasticity concept has been widely accepted and has been applied to many problems of a wide range of interest, including the bending, buckling, and vibration of beam-like (Aydogdu, 2009; Liu et al., 2008; Reddy, 2007) and plate-like (Narendar, 2011; Pradhan and Phadikar, 2009; Shen et al., 2010) elements in micro/nano structures.

The application of nonlocal elasticity models in micro/nano materials was initially delivered by Peddieson et al. (2003) in which a nonlocal version of Euler–Bernoulli beam model was formulated based on the nonlocal elasticity theory of Eringen (1983). Up to now, many research works correlated to nonlocal elasticity theory have been reported trying to develop nonlocal continuum models and apply them to analyze the general behavior of structures in micro/nano scale, see Refs. (Lu et al., 2006; Wang et al., 2008; Murmu and Pradhan, 2009; Civalek and Akgöz, 2010; Civalek and Demir, 2011; Setoodeh et al., 2011; Thai, 2012; Eltaher et al., 2013; Rahmani and Pedram, 2014) and the references therein. It has been clear from these available surveys that nonlocal effects play a significant role both for static and dynamic issues exhibited by micro/nano structures.

It is fairly necessary to perform the nonlinear vibration analysis of micro/nano beams in thermal environments. Firstly, for many micro/nano beams, both the physical (e.g., induced by van der Waals force) and geometrical (e.g., stem from large deflection) nonlinearities were observed by previous theoretical and experimental investigations (Yang et al., 2010; Ke et al., 2009), the nonlinearity causes the mechanical behaviors of the beams to be changed significantly (Setoodeh et al., 2011; Fang et al., 2013), so the linear beam theory is not appropriate in such situations. Moreover, some researches confirm that the thermal effects are effective on the mechanical behaviors of micro/nano beams (Amara et al., 2010; Ansari et al., 2011). Based on the theories of thermal elasticity and nonlocal elasticity, Chang (2012) developed an elastic Euler-Bernoulli beam model for the thermal-mechanical vibration and buckling instability of single-walled carbon nanotubes conveying fluid, numerical solutions obtained from the finite element method conclude that the effects of temperature change and nonlocal small scale are very significant on the fundamental natural frequency and critical flow velocity. Janghorban (2012) investigated the bending of an Euler–Bernoulli microbeam in thermal environment based on nonlocal elasticity theory using two types of differential quadrature method to discretize the equilibrium equation. Yang and Lim (2012) derived a new higher-order nonlocal Timoshenko beam model for thermal buckling of a shear deformable nanocolumn via the variational principle and von kármán nonlinearity. Size-dependent thermal buckling behavior of nanocolumns was demonstrated. Arani et al. (2012), Ke et al. (2012) investigated the nonlinear vibration of the piezoelectric nanobeams based on the nonlocal theory and Timoshenko beam theory under the combination of electric and thermal fields. The nonlinear governing equations were derived by using Hamilton principle and the numerical solutions for the nonlinear frequency were determined employing differential quadrature method. Nazemnezhad and Hosseini-Hashemi (2014) studied the nonlinear free vibration of functionally graded nanobeams within the framework of nonlocal elasticity and Euler–Bernoulli beam theory with von kármán type nonlinearity. The approximate analytical expression for nonlinear frequency was established utilizing the Galerkin method and multiple scale method. Similar study was carried out by Şimşek (2014) for nanobeams, the nonlinear vibration frequency was obtained in analytical form via the Galerkin method and a variational method.

As one may note, the majority of cited references on the vibration modeling of micro/nano beams are confined to linear cases with or without thermal effect incorporated. Relatively limited attempt is available related to the large amplitude vibration behavior of micro/nano-scale beams in an environment of changing temperatures. That gives us a potential to investigate the nonlinear vibration of micro/nano beams by nonlocal thermal elasticity theory.

This paper makes the effort to explore the influence of Von Kármán geometric nonlinearity on the vibration behavior of a micro/nano beam in accordance with the Euler-Bernoulli beam theory with inclusion of the thermal environment. The small scale effect is taken into consideration based on the nonlocal elasticity theory. The governing equations are derived adopting Hamilton principle and the numerical solution for the vibration frequency is obtained making use of the Kantorovich time-averaging method (Huang and Aurora, 1979; Huang and Walker, 1988) followed by the shooting method (Li and Zhou, 2001; Wang et al., 2013; William et al., 1986). The effects of geometric nonlinearity, aspect ratio (length-to-depth ratio), temperature rise and nonlocal parameter on the vibrational frequency are studied in detail. From the knowledge of authors, it is the first time using the shooting method for the nonlocal beam vibrating in the nonlinear regime and temperature field. It is believed that the present model can be a promising technique to offer an efficient and accurate nonlinear vibration solution in investigating micro/nano beams with nonlocal effects.

2 GOVERNING EQUATIONS OF MICRO/NANO BEAMS

The beam under consideration is modeled as an Euler-Bernoulli one at micro/nano scale. It has length L, height (or depth) h, rectangular cross section area A, and moment of inertia I. The Cartesian coordinate system (x, y, z) is used with x-axis coincident with the centroidal axis of the undeformed beam, y-axis along the width, and z-axis along the height of the beam.

2.1 Strain-displacement relations

Assuming that the deformations of the beam take place in the x-z plane. Upon denoting the displacement components along the x-, y- and z-directions by u_1, u_2, and u_3, respectively, the displacement field can be written in view of the assumptions of Euler-Bernoulli beams as (Wang et al., 2013)

$$u_1(x,z,t) = u(x,t) - zw_{,x}(x,t), \quad u_2(x,z,t) = 0, \quad u_3(x,z,t) = w(x,t) \tag{1}$$

where $x \in [0, L]$, t is the time variable, $w(x,t)$ and $u(x,t)$ are the displacements of the neutral axis at abscissa x in z- and x-directions, respectively. Hereinbelow, a comma represent the partial derivative with respect to the indicated spatial and temporal coordinates.

In accordance, the von Kármán type nonlinear strain–displacement relationship takes the following form

$$\varepsilon_{xx}(x,z,t) = u_{,x} + \frac{1}{2}w_{,x}^2 - zw_{,xx} \tag{2}$$

2.2 Nonlocal beam theory and stress resultants

For the case where the thermal effect is taken into account, the nonlocal constitutive relation can be approximated to a one-dimensional form as (Eringen, 1983 and 2002)

$$\chi(\sigma_{xx}) = E\varepsilon_{xx} - E\alpha T \tag{3}$$

where E is the elastic modulus, α is the thermal expansion coefficient and T denotes the temperature rise; σ_{xx} is the normal stress, ε_{xx} is the normal strain, respectively. χ is a nonlocal linear differential operator defined by

$$\chi(\ ) = 1 - \mu^2(\ )_{,xx}, \quad \mu = e_0 a \tag{4}$$

in which μ is the nonlocal parameter that incorporates the small scale effect, e_0 is a constant appropriate to each material, and a is an internal characteristic length.

The nonlocal constitutive relation can be expressed in terms of stress resultants. Integrating Eq.(3) over the beam's cross section area A yields

$$\chi(N_{xx}) = EA\left(u_{,x} + \frac{1}{2}w_{,x}^2\right) - N^T \tag{5}$$

Multiplying Eq.(3) by z and integrating the result over the area A leads

$$\chi(M_{xx}) = -EIw_{,xx} - M^T \tag{6}$$

where the moment of inertia I, the stress resultants N_{xx} and M_{xx}, and the thermal stress resultants N^T and M^T, are, respectively, defined by

$$(A, I) = \int_A (1, z^2)\,\mathrm{d}A, \quad (N_{xx}, M_{xx}) = \int_A (1, z)\sigma_{xx}\mathrm{d}A, \quad (N_{xx}, M_{xx}) = \int_A (1, z)\sigma_{xx}\mathrm{d}A$$

2.3 Nonlinear equations of motion

The dynamic behavior of the Euler-Bernoulli beam is governed by Hamilton principle which can be stated in analytical form as

$$\int_{t_1}^{t_2}\left(\delta \Pi - \delta K_E - \delta V\right)\mathrm{d}t = 0 \tag{7}$$

where Π is the strain energy, K_E the kinetic energy, and V the work done by the external applied forces.

The first variation of strain energy of the beam takes the form

$$\delta \Pi = \int_0^L \int_A \sigma_{xx}\delta\varepsilon_{xx}\,\mathrm{d}A\mathrm{d}x = \int_0^L \left[N_{xx}\left(\delta u_{,x} + w_{,x}\delta w_{,x}\right) - M_{xx}\delta w_{,xx}\right]\mathrm{d}x \tag{8}$$

The first variation of kinetic energy states that

$$\delta K_E = \int_0^L \left[\rho A\left(u_{,t}\delta u_{,t} + w_{,t}\delta w_{,t}\right) + \rho I w_{,xt}\delta w_{,xt}\right]\mathrm{d}x \tag{9}$$

an expression in which the longitudinal and rotational inertia are included. Here, ρ is the mass density of the beam material.

Denoting q_u and q_w be the external axial and transverse loads distributed along the length of beam, respectively, then the first variation of the work done by the external forces reads

$$\delta V = \int_0^L \left(q_u\delta u + q_w\delta w\right) \tag{10}$$

Substituting Eqs.(8)-(10) for $\delta\Pi$, δK_E, and δV into Eq.(7), integrating by parts and grouping terms by δw and δu lead to the following partial differential equations

$$\int_{t_1}^{t_2}\int_0^L \left[\rho A u_{,tt} - N_{xx,x} - q_u\right]\delta u\,\mathrm{d}x\,\mathrm{d}t = 0 \tag{11}$$

$$\int_{t_1}^{t_2}\int_0^L \left[\rho A w_{,tt} - \rho I w_{,xxtt} - M_{xx,xx} - \left(N_{xx}w_{,x}\right)_{,x} - q_w\right]\delta w\,\mathrm{d}x\,\mathrm{d}t = 0 \tag{12}$$

and the corresponding boundary conditions at the beam ends

$$\int_{t_1}^{t_2} N_{xx}\delta u\Big|_{x=0,L}\mathrm{d}t = 0,\quad \int_{t_1}^{t_2} V_x\delta w\Big|_{x=0,L}\mathrm{d}t = 0,\quad \int_{t_1}^{t_2} M_{xx}\delta w_{,x}\Big|_{x=0,L}\mathrm{d}t = 0 \tag{13}$$

with V_x denotes the equivalent shear force

$$V_x = N_{xx}w_{,x} + M_{xx,x} + \rho I w_{,xtt} \tag{14}$$

Noting that δw and δu are arbitrary, setting the coefficients of δw and δu in Eqs.(11)-(12) to zero, and substituting for the second derivative of N_{xx} and M_{xx} from the subsequent results into Eqs.(5)-(6) and Eq.(14), the explicit expressions of nonlocal stress resultants and V_x are obtained as

$$N_{xx} = EA\left(u_{,x} + \frac{1}{2}w_{,x}^2\right) - N^T + \mu^2\left(\rho A u_{,tt} - q_u\right)_{,x} \tag{15}$$

$$M_{xx} = -EIw_{,xx} - M^T + \mu^2\left(\rho A w_{,tt} - \rho I w_{,xxtt} - \left(N_{xx}w_{,x}\right)_{,x} - q_w\right) \tag{16}$$

$$V_x = -EIw_{,xxx} - M_{,x}^T + \chi\left(N_{xx}w_{,x} + \rho I w_{,xtt}\right) + \mu^2\left(\rho A w_{,tt} - q_w\right)_{,x} \tag{17}$$

As a final point, the nonlocal governing equations in terms of the displacements can be expressed by substituting for N_{xx} and M_{xx} from Eqs.(15)-(16), respectively, into Eqs.(11)-(12) as follows

$$\int_{t_1}^{t_2}\int_0^L\left[EA\left(u_{,x} + \frac{1}{2}w_{,x}^2\right)_{,x} - N_{,x}^T + \chi\left(q_u - \rho A u_{,tt}\right)\right]\delta u \,\mathrm{d}x\,\mathrm{d}t = 0 \tag{18}$$

$$\int_{t_1}^{t_2}\int_0^L\left\{EIw_{,xxxx} + M_{,xx}^T + \chi\left[\rho A w_{,tt} - \rho I w_{,xxtt} - \left(N_{xx}w_{,x}\right)_{,x} - q_w\right]\right\}\delta w \,\mathrm{d}x\,\mathrm{d}t = 0 \tag{19}$$

For the subsequent results to be general, the following parameters are defined to make the governing equations dimensionless

$$\left(X,\ U,\ W\right) = \frac{1}{L}\left(x,\ u,\ w\right),\ \kappa = L\sqrt{\frac{A}{I}},\ \tau = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}},\ \eta = \frac{\mu}{L},\ \tilde{\chi}(\ \ ) = 1 - \eta^2(\ \ )_{,XX}$$

$$\left(\bar{M},\ \bar{M}^T\right) = \frac{L}{EI}\left(M_{xx},\ M^T\right),\ \left(P_H,\ P_V,\ \bar{N}^T\right) = \frac{L^2}{EI}\left(N_{xx},\ V_x,\ N^T\right),\ \left(Q_u,\ Q_w\right) = \frac{L^3}{EI}\left(q_u,\ q_w\right)$$

where κ is the slenderness ratio of the beam, η is the scaling effect parameter. By using these variables, the nonlocal governing equations as well as the boundary conditions can be rewritten in the following form

$$\int_{\tau_1}^{\tau_2}\int_0^1\left[\kappa^2\left(U_{,X} + \frac{1}{2}W_{,X}^2\right)_{,X} - \bar{N}_{,X}^T - \tilde{\chi}\left(U_{,\tau\tau} - Q_u\right)\right]\delta U\,\mathrm{d}X\,\mathrm{d}\tau = 0 \tag{20}$$

$$\int_{\tau_1}^{\tau_2}\int_0^1\left\{W_{,XXXX} + \tilde{\chi}\left[W_{,\tau\tau} - \frac{1}{\kappa^2}W_{,XX\tau\tau} - (P_H W_{,X})_{,X} - Q_w\right] + \bar{M}_{,XX}^T\right\}\delta W\,\mathrm{d}X\,\mathrm{d}\tau = 0 \tag{21}$$

$$\int_{\tau_1}^{\tau_2} P_H\delta U\big|_{X=0,1}\mathrm{d}\tau = 0,\quad \int_{\tau_1}^{\tau_2} P_V\delta W\big|_{X=0,1}\mathrm{d}\tau = 0,\quad \int_{\tau_1}^{\tau_2}\bar{M}\delta W_{,X}\big|_{X=0,1}\mathrm{d}\tau = 0 \tag{22}$$

in which

$$P_H = \kappa^2\left(U_{,X} + \frac{1}{2}W_{,X}^2\right) - \bar{N}^T + \eta^2\left(U_{,\tau\tau} - Q_u\right)_{,X} \tag{23}$$

$$P_V = -W_{,XXX} - \bar{M}_{,X}^T + \tilde{\chi}\left(P_H W_{,X} + \frac{1}{\kappa^2}W_{,X\tau\tau}\right) + \eta^2\left(W_{,\tau\tau} - Q_w\right)_{,X} \tag{24}$$

$$\bar{M} = -W_{,XX} - \bar{M}^T + \eta^2\left[W_{,\tau\tau} - \frac{1}{\kappa^2}W_{,XX\tau\tau} - (P_H W_{,X})_{,X} - Q_w\right] \tag{25}$$

The integro-differential Eqs.(20)-(21) along with the boundary conditions (22) govern the von Kármán nonlinearity of the proposed beam with allowances for small scale effect. As a remark, the proposed nonlocal theory can be straightforwardly addressed to the nonlocal beam for bending, thermal buckling and dynamic analyses. A notable feature of present model is that both the governing equations and boundary conditions are nonlocal due to the nonlocal constitutive relations. Furthermore, it is worth emphasizing that conventional (local) beam model is recovered when the influences of small scale is disregarded.

Herein, a pinned-pinned beam will be considered. In view of the symmetrical deformation about the center of the beam, a half beam is modeled and the boundary conditions at the ends are of the following form based on Eq.(22)

$$W\left(0,\tau\right) = 0, \quad U\left(0,\tau\right) = 0, \quad \bar{M}\left(0,\tau\right) = 0 \tag{26}$$

$$W_{,X}\left(\frac{1}{2},\tau\right) = 0, \quad U\left(\frac{1}{2},\tau\right) = 0, \quad P_V\left(\frac{1}{2},\tau\right) = 0 \tag{27}$$

3 METHOD OF SOLUTION

The current work centers on the nonlinearly free vibration of a uniformly heated nonlocal beam with time-independent temperature. So the basic equations should be modified by setting

$$Q_w = Q_u = \bar{M}^T = 0, \quad \bar{N}^T = \kappa^2\alpha T = \lambda \tag{28}$$

with λ being a parameter introduced to indicate the temperature rise.

Then, in consideration of Eq.(20) cum Eq.(23), the dimensionless relationship between the axial stress resultant and displacement reads

$$\int_{\tau_1}^{\tau_2}\int_0^1\left(P_{H,x} - U_{,\tau\tau}\right)\delta U \,\mathrm{d}X\,\mathrm{d}\tau = 0 \tag{29}$$

an expression that will be used in the numerical procedure to simplify the computation.

The partial differential governing equations and boundary conditions (20)-(22) are complicated due to the nonlinearity and coupling between the longitudinal and transverse displacements. A closed-form analytical solution is difficult to obtain. In solving the nonlinear free vibration, as a powerful numerical technique, the combination of Kantorovich averaging method (Huang and Aurora, 1979; Huang and Walker, 1988) and the shooting method (Li and Zhou, 1988; Wang et al., 2013; William et al., 1986) is frequently applied to treat relevant problems. Advantage of this approach lies in its ease to handle more complex nonlinear boundary value problem. However, to best knowledge of authors, the approach is not available in the open literature for the nonlocal elastic structures. The method for the problem at hand is applied in what follows.

3.1 Kantorovich averaging method

Assuming that the vibration is prior to the buckling of the beam, that is, $\lambda \le \lambda_{cr}$, where λ_{cr} is the critical temperature rise parameter. Major concern is focused on the microstructure-dependent characteristic relation between the fundamental frequency and vibrational displacement of a beam. Approximate solutions of large amplitude free vibration will be achieved by implementing the following assumed harmonic temporal functions (Wang et al., 2013)

$$W\left(X,\tau\right) = \tilde{W}\left(X\right)\cos\omega\tau, \quad U\left(X,\tau\right) = \tilde{U}\left(X\right)\cos^2\omega\tau \tag{30}$$

where ω is a nondimensional radian frequency of the beam, $\tilde{W}\left(X\right)$ and $\tilde{U}\left(X\right)$ are the shape functions to be determined.

The Kantorovich time-averaging method is then applied to eliminate the temporal coordinate. The substitution of Eq.(30) into Eqs.(20)-(21) cum Eq.(29), and the integration over one complete period of oscillation, $0 \sim 2\pi/\omega$, lead to the following set of nonlinear ordinary differential equations

$$\frac{\mathrm{d}\mathbf{Y}}{\mathrm{d}X} = \mathbf{H}\left(X,\mathbf{Y};\beta,\lambda,\eta\right), \quad X \in \left(0,1/2\right) \tag{31}$$

with the denotations of the forms

$$\mathbf{Y} = \left\{Y_1,Y_2,Y_3,Y_4,Y_5,Y_6,Y_7\right\}^{\mathrm{T}} = \left\{\tilde{W},\tilde{W}_{,X},\tilde{W}_{,XX},\tilde{W}_{,XXX},\tilde{U},\tilde{U}_{,X},\omega^2\right\}^{\mathrm{T}}$$

$$\mathbf{H} = \left\{Y_2,Y_3,Y_4,\lambda\!\!\!^{-}_1\varphi_1,Y_6,\lambda\!\!\!^{-}_2\varphi_2,0\right\}^{\mathrm{T}}$$

$$\varphi_1 = N_sY_3 - Y_7\left(Y_2Y_5 - Y_1 + \frac{1}{\kappa^2}Y_3\right) + \eta^2Y_7\left(3Y_3Y_6 + 3Y_4Y_5 + \lambda\!\!\!^{-}_2\varphi_2Y_2 - Y_3\right), \ \varphi_2 = 3\kappa^2Y_2Y_3 + 4Y_5Y_7$$

$$\lambda\!\!\!^{-}_1 = \frac{1}{1+\eta^2N_s - Y_7\eta^2/\kappa^2}, \ \lambda\!\!\!^{-}_2 = \frac{1}{4\eta^2Y_7 - 3\kappa^2}, \ N_s = \frac{3}{4}\kappa^2\left(Y_6 + \frac{1}{2}Y_2^2\right) - \eta^2Y_6Y_7 - \lambda$$

Since Eq.(30) cannot satisfy all the boundary conditions in Eqs.(26)-(27) identically, a residual may exist. The execution of the Kantorovich method to Eqs.(26)-(27) in virtue of Eq.(22), yields a group of scale-dependent boundary conditions. However, in the case of buckling and vibration analyses, the nonlocal bending moment $\bar{M} = 0$ in Eq.(26) can be replaced by $\tilde{W}_{,XX} = 0$ as a result of $\tilde{W} = \tilde{U} = 0$ at the pinned end, and the nonlocal shear force $P_V = 0$ in Eq.(27) can be replaced by $\tilde{W}_{,XXX} = 0$ due to $\tilde{W}_{,X} = \tilde{U} = 0$ at the mid-span of the beam. This leads the boundary conditions are the same as those pertaining to the classical beam theory and can be expressed in shape functions as

$$\mathbf{B}_1\mathbf{Y}(0) = \left\{Y_1, Y_5, Y_3\right\}^{\mathrm{T}} = \mathbf{0},\ \ \mathbf{B}_2\mathbf{Y}\left(\frac{1}{2}\right) = \left\{Y_1, Y_2, Y_5, Y_4\right\}^{\mathrm{T}} = \left\{\beta, 0, 0, 0\right\}^{\mathrm{T}} \tag{32}$$

Here, $\beta = W\left(1/2, 0\right) = \tilde{W}\left(1/2\right)$ indicates the normalized central amplitude of the beam, $\mathbf{B}_1$ and $\mathbf{B}_2$ are two boundary related matrixes.

3.2 Brief description of shooting method (William et al., 1986)

Eqs.(31)-(32) constitute a nonlinear spatial boundary value problem including fundamental frequency parameter ω, scaling effect parameter η, and temperature rise parameter λ. The shooting method, consisting of the Runge-Kutta integration method in conjunction with the Newton-Raphson iterative formulation, is employed to numerically get a solution of the problem.

To apply the shooting algorithm, the boundary value problem (31)-(32) is first reduce to an initial one as

$$\frac{\mathrm{d}\mathbf{Z}}{\mathrm{d}X} = \mathbf{H}\left(X, \mathbf{Z}; \beta, \lambda, \eta\right), \quad X > 0 \tag{33}$$

$$\mathbf{Z}(0) = \left\{0, d_1, 0, d_2, 0, d_3, d_4\right\}^{\mathrm{T}} \tag{34}$$

where $\mathbf{Z} = \left\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\right\}^{\mathrm{T}}$, and $\mathbf{D} = \left\{d_1, d_2, d_3, d_4\right\}^{\mathrm{T}}$ is an unknown vector related to the missing initial values of $\mathbf{Y}$ at $X = 0$. Integrating Eq.(33) with Eq.(34) one calculates $\mathbf{Z}$. A fourth order Runge-Kutta method with variable steps may be used with this purpose.

After prescribing parameters, β, λ, and η, the Newton-Raphson formulation is applied and $\mathbf{D}$ is updated in a way that the answers satisfy the four finial conditions at $X = 1/2$, namely,

$$\mathbf{B}_2\mathbf{Z}\left(\frac{1}{2}; \beta, \lambda, \eta, \mathbf{D}\right) = \left\{\beta, 0, 0, 0\right\}^{\mathrm{T}} \tag{35}$$

According to the iterative procedure, if convergence is achieved, or if $\mathbf{D} = \mathbf{D}^*$ is a approximate answer of equation (35), the correct value for boundary value problem can be determined

$$\mathbf{Y}\left(X; \beta, \lambda, \eta\right) = \mathbf{Z}\left(X; \beta, \lambda, \eta, \mathbf{D}^*\right) \tag{36}$$

which contains the frequency dependence as

$$\omega^2 = Y_7\left(\beta, \lambda, \eta\right) \tag{37}$$

4 NUMERICAL RESULTS AND DISCUSSION

Imposing the formulation and algorithm outlined in the previous section, some numerical investigations are now performed and discussed. The nondimensional fundamental frequencies are presented in both tabular and graphical forms with varying nonlocal parameter, aspect ratio, temperature load, and central amplitude. To facilitate a direct comparison with the existing data available in the literature, the dimensionless amplitude at the center of the beam is sometimes normalized by $\kappa\beta$.

4.1 Comparison with published results

In order to establish reasonable comparisons, the linear free vibration of a nanobeam with its length L is assumed to be 10 nm is examined for various values of nonlocal parameter μ^2 . Here, the linear vibration can be recovered by setting the amplitude parameter β to be a very small value. At first, the natural fundamental frequencies ω_0 of the beam with some cases of aspect ratio L/h are tabulated in Table 1 in the absence of the temperature rise λ and are compared with the results given by Thai (2012). Excellent agreement is observed.

μ^2 (nm^2)	ω_0 when $L/h=5$		ω_0 when $L/h=10$		ω_0 when $L/h=20$		ω_0 when $L/h=100$		λ_{cr} when $L/h=20$	
	Present	Thai (2012)	Present	Thai (2012)	Present	Thai (2012)	Present	Thai (2012)	Present	Thai (2012)
0	9.7112	9.7112	9.8293	9.8293	9.8595	9.8595	9.8692	9.8692	9.8696	9.8696
1	9.2647	9.2647	9.3774	9.3774	9.4062	9.4062	9.4155	9.4155	8.9830	8.9830
2	8.8747	8.8747	8.9826	8.9826	9.0102	9.0102	9.0192	9.0191	8.2426	8.2426
3	8.5301	8.5301	8.6338	8.6338	8.6603	8.6604	8.6689	8.6689	7.6149	7.6149
4	8.2228	8.2228	8.3228	8.3228	8.3483	8.3483	8.3567	8.3566	7.0761	7.0761

Table 1: Comparison of the natural radian frequency parameter ω_0 and the critical temperature rise parameter λ_{cr} for various nonlocal parameters of a nanobeam.

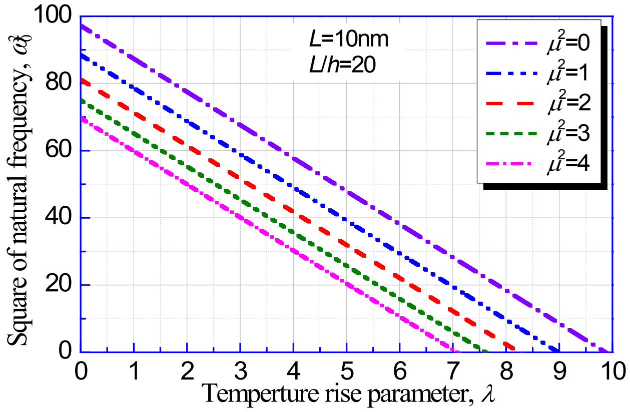

Figure 1: Variation of the square of natural fundamental frequency with the temperature rise parameter under some specific values of nonlocal parameter.

Then verification is carried out for the linear vibration problem of a uniformly heated beam. The dependences of the square of fundamental frequency ω_0^2 on the temperature rise parameter λ under some specific values of nonlocal parameter μ^2 at the aspect ratio L/h are plotted in Fig. 1, in which, an unheated beam result is rendered when λ vanishes, and a classical beam result is recovered when μ^2 is identically zero. The point with $\omega_0 = 0$ matches with the critical temperature rise parameter λ_{cr}, which is extracted from Fig. 1, and is listed in Table 1 for comparison with Thai (2012). It is clearly understood from the curves shown in Fig. 1 that the inclusion of the nonlocal effects decreases the buckling temperature rise as well as the natural frequency. In other words, the local theory overestimates the buckling temperature and natural frequency of the beam compared to the nonlocal one. For a given μ, the square of fundamental frequency decreases monotonically and almost linearly with the increment of the temperature rise parameter, this trend is in agreement with that reported in the literature for a heated plate (Li and Zhou, 2001).

Furthermore, the reliability of the present method in nonlinear free vibration analysis is demonstrated for an unheated beam with specified scaling effect parameters η. The normalized fundamental frequency defined by the ratio of the nonlinear to linear radian frequency parameter ω/ω_0 is presented in Table 2 at varying vibration amplitude $\kappa\beta$ and is compared with the one obtained by Şimşek (2014). It may be noted that the present results match quite well to those of the other researchers, validating the present computational techniques.

$\kappa\beta$	$\eta = 0$		$\eta = 0.2$		$\eta = 0.4$		$\eta = 0.6$		$\eta = 0.8$	
	Present	Şimşek (2014)	Present	Şimşek (2014)	Present	Şimşek (2014)	Present	Şimşek (2014)	Present	Şimşek (2014)
0.5	1.02322	1.02316	1.03225	1.03217	1.05887	1.05872	1.10182	1.10155	1.15926	1.15886
1.0	1.08981	1.08972	1.12330	1.12317	1.21827	1.21802	1.36191	1.36150	1.54067	1.54008
1.5	1.19250	1.19242	1.26046	1.26032	1.44529	1.44501	1.70950	1.70904	2.02219	2.02155
2.0	1.32291	1.32287	1.43052	1.43041	1.71326	1.71299	2.10158	2.10114	2.54765	2.54703
2.5	1.47368	1.47372	1.62316	1.62311	2.00581	2.00559	2.51744	2.51706	3.09473	3.09420
3.0	1.63917	1.63935	1.83125	1.83131	2.31363	2.31350	2.94703	2.94674	3.65373	3.65330
3.5	1.81536	1.81572	2.05009	2.05027	2.63136	2.63134	3.38506	3.38493	4.21983	4.21962
4.0	1.99941	2.00000	2.27657	2.27691	2.95576	2.95591	3.82869	3.82872	4.79058	4.79057

Table 2: Comparison of the normalized fundamental frequency ω_0/ω_0 for various dimensionless central amplitudes and scaling effect parameters of a nanobeam.

4.2 Nonlocal amplitude frequency dependence

At this stage, the parametric studies are conducted to demonstrate the effects of nonlocality, temperature rise, central amplitude, and aspect ratio on the fundamental frequency of micro/nano beams. For a clear manifestation, numerical results are presented graphically.

The variation of fundamental frequency with varying central amplitude under some assigned values of temperature rise and scaling effect parameter at the aspect ratio is chosen as $L/h = 20$ is demonstrated in Fig. 2, in which, the linear frequency is recovered when $\kappa\beta = 0$, and a local frequency is achieved when $\eta = 0$. Pronounced influences of the temperature rise and nonlocality on the nonlinear vibration behavior are observed from this figure. It is found that the relationship of amplitude-frequency is of the hardening type, and the increment in small scale and temperature rise leads to a reduction of the vibration frequency due to the reduction in the flexural rigidity of the beam. In addition, the temperature and nonlocality effects are more obvious for lower central amplitude and they decrease with increasing amplitude.

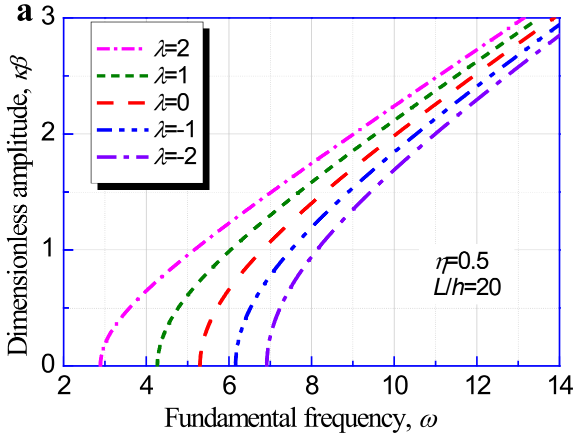

Figure 2: Variation of the fundamental frequency with the central amplitude under (a) various temperature rise parameters and (b) various scaling effect parameters.

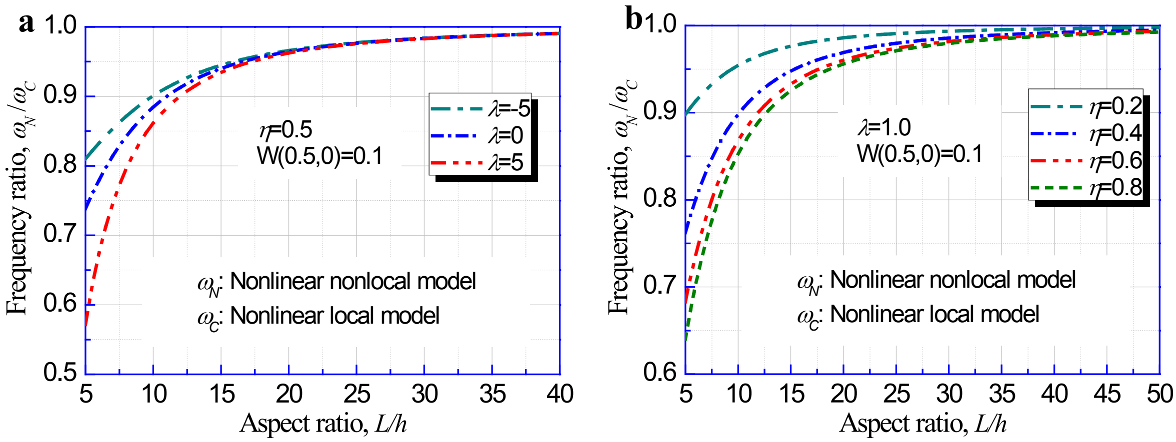

Figure 3: Frequency ratio of the nonlocal fundamental frequency to the local one vs. aspect ratio for a beam under (a) various temperature rise parameters and (b) various scaling effect parameters.

It is expected that the small scale effect will diminish for a very slender beam. Fig. 3 confirms this point by showing the variation of the fundamental frequency ratio versus the aspect ratio for $\beta = W(0.5, 0) = 0.1$ with different temperature rise and scaling effect parameter. Here, the frequency ratio is defined by the ratio of the nonlocal fundamental frequency to the local counterpart. The ratio so defined is often used as an index to assess quantitatively the small scale effect on micro/nano-size structures vibration in nonlocal elasticity theory. It can be seen that the frequency ratio is less than unity for all aspect ratios, indicating that the frequency calculated using local theory will be the largest one and the inclusion of effect of scale coefficient leads to a reduction in the vibration frequency compare to that excluding the effect of scale. With increasing aspect ratio L/h, results are converging to unity, the frequency gradually approaches the local limit. So, it is inferred that nonlocality is important for lower L/h and is negligible for higher L/h. Moreover, the nonlocality and temperature effects on frequency ratios become more by increasing their values.

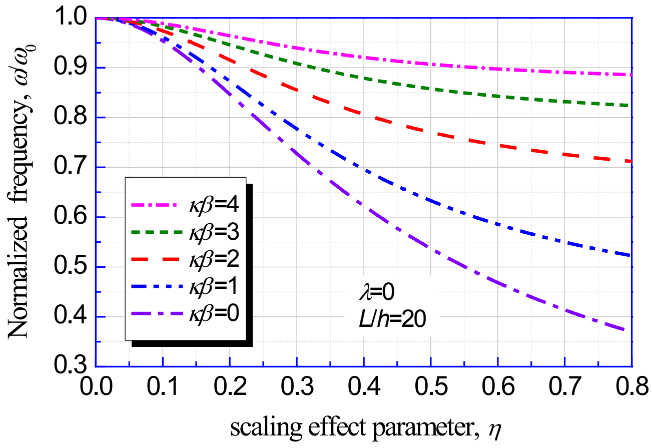

Figure 4: Normalized fundamental frequency vs. scaling effect parameter under some dimensionless central amplitude.

The dependency of the normalized fundamental frequency on scaling effect parameter for some selected values of the dimensionless amplitude is showed in Fig. 4. According to Fig. 4, it is observed that the nonlocal parameter causes a reduction while the amplitude parameter causes a rise in the frequency ratio. A point to note is that, for frequency ratio at lower amplitude, the nonlocal parameter shows more decreasing effect. Further-more, as the nonlocal parameter increases, the frequency ratio becomes far from unity which highlights the significance of the nonlocal effect. Also, fixing the scaling effect parameter η and varying the dimensionless amplitude parameter $\kappa\beta$ results in a significant change in the frequency ratio, revealing the amplitude dependency of the degree of nonlinearity.

5 CONCLUSIONS

The large amplitude free vibration analysis for an Euler-Bernoulli micro/nano beam model has been pursued in thermal environments exploiting the nonlocal constitutive equation to describe the scaling effect. The governing equations are derived by Hamilton principle and the size-dependent frequency is determined based on the assumed time-mode method, Kantorovich method, and shooting technique. It is shown that the numerical results obtained by the proposed formulations and algorithm coincide perfectly with the previous results. The effects of nonlocal parameter, aspect ratio, temperature rise and central amplitude on the nonlinear vibration frequency are investigated.

Parametric study has been presented where the frequency obtained for the nonlocal theory is less than the one for its local counterpart. The decrease in frequency is attributed to the reduction of stiffness of the beam due to the nonlocality. Therefore, it can be inferred that application of the local models for micro/nano-sized beam would lead to an overprediction of the frequency.

The frequency difference between local and nonlocal theories is more significant for high value of the nonlocal parameter.

It seems that the prebuckling temperature rise plays a similar action to that of the nonlocal parameter, the decreasing trend of the frequency parameter has been demonstrated with increasing temperature rise. This decrease is owing to the reduction of flexural rigidity due to the compressive in-plane load induced by the temperature rise.

The critical temperature rise at which buckling occurs can be extracted as a byproduct through setting the fundamental natural frequency to zero.

The nonlinear vibration frequency gradually converges to its local value as the aspect ratio becomes large.

For vanishing nonlocal parameter, the vibration solution for a beam furnished by the classical elasticity theory is recovered, and for vanishing temperature rise, the vibration solution for a unheated beam is degenerated.

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