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A Note on the Inverse Reconstruction of Residual Fields in Surface Peened Plates

Abstract

A modified stress function approach is developed here to reconstruct induced stress, residual stress and eigenstrain fields from limited experimental measurements. The present approach is successfully applied to three experimental measurements set in surface peened plates with shallow shot peening affected zone. The well-rehearsed advantage of the proposed approach is that it not only minimizes the deviation of measurements from its approximations but also will result in an inverse solution satisfying a full range of continuum mechanics requirements. Also, the effect of component thickness as a geometric parameter influencing the residual stress state is comprehensively studied. A key finding of present study is that the plate thickness has no influence on the maximum magnitude of eigenstrain profile and compressive residual stresses within the shot peening affected zone while having a great influence on the magnitude of tensile residual stress and the gradient of linear residual stresses present in deeper regions.

Keywords

Surface peening, Residual stress, Induced Stress, Inverse problems, Eigenstrain.

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1 INTRODUCTION

Shot peening is a mechanical surface treatment generally utilized to improve the stress corrosion resistance and enhance the fatigue life of engineering structures (Meguid, 1991). It is well known that the shot peening process induces compressive residual stresses and work hardening in near-surface regions of treated components in order to suppress and decelerate crack initiation and propagation in engineering components subjected to cyclic loading. Therefore predicting the distribution and magnitude of residual stresses generated by shot peening is of utmost importance in understanding the influence of physical parameters of the process (Fathallah et al., 1998).

A remarkable set of publications could be found in the literature dedicated to the modeling of residual stress state of shot peening process utilizing different types of finite element modeling (Guagliano, 2001; Zimmermann et al., 2009; Gonzales et al., 2010), analytical methods (Shen and Atluri, 2006; Franchim et al., 2009; Miao et al., 2010) or experimental methods (Niku-Lari et al., 1985; Fathallah et al., 1998; Menig et al., 2001). However it is currently difficult and prohibitively time-consuming to measure the complete patterns of residual stresses throughout the most structural materials (Krawitz, 2011). Consequently most measurements are restricted and hence resulted in relatively incomplete experimental information. So, there is great practical value in inferring the complete residual stress field from a limited number of measurements (Coules et al., 2014).

Theory of inverse method for determination of residual stress field from limited experimental measurements has been developed over the past decade and gained a great attention. One such method known as variational eigenstrain method is based on the theory of eigenstrains. The variational eigenstrain method utilized a combination of finite element analysis and distributed basis eigenstrains that combines experimental characterization in terms of residual elastic strains (Korsunsky et al., 2007; Korsunsky, 2009; Jun and Korsunsky, 2010). A smoothed inverse eigenstrain method is also developed for reconstruction of residual field from limited strain measurements that allows suppressing fluctuations that are contrary to the physics of the problem (Faghidian, 2014). An alternative method is the stress function approach which does not utilize assumed eigenstrain distribution. Stress function approach approximate the residual stress field using a series of stress functions which directly solve the stress equilibrium equations together with the traction free boundary conditions (Farrahi et al., 2009a, 2010; Faghidian et al., 2012). Reconstruction of residual stress distribution due to surface peening was first attempted by Korsunsky (2005) utilizing the variational eigenstrain approach. Afterward based on the stress function approach, axi-symmetric residual stress field was reconstructed in shot-blasted steel round bars (Farrahi et al., 2009b). Also Jun et al. (2011) analyzed residual elastic strains in non-uniformly shaped shot-peened samples based on the variational eigenstrain approach. Recently based on the stress function approach, Faghidian (2015) reconstruct the residual fields form experimental measurements in surface peened plates with deep shot peening affected zone utilizing Cauchy type modulation functions. Also Faghidian (2015) performed a full discussion on various mathematical aspects of numerical reconstruction including the regularity issue of the approximated solution by Tikhonov-Morozov regularization method.

In present study, the stress function approach is modified to reconstruct the residual stress and eigenstrain field form experimental residual stress measurements in surface peened plates with shallow shot peening affected zone. The proposed approach results in a continuous reconstructed residual stresses and eigenstrains while satisfying all of the continuum mechanics requirements. In contrast to stress function approach where the residual stress field are directly determined from Airy stress function, in present approach the induced stress field would be first reconstructed utilizing Gaussian type modulation function. Subsequently the residual stress distribution and eigenstrain field are inversely determined satisfying the static balance across the domain and strain compatibility equations, respectively. Furthermore, the effect of component thickness as a geometric parameter influencing the residual stress state and eigenstrain distribution would be examined.

2 DETERMINATION OF RESIDUAL FIELDS

2.1 Residual stresses

Residual stresses are the stress field supported by a continuum in a fixed reference configuration where there is no external forces and thermal gradients (Hoger, 1986). The domain of interest containing the distribution of residual stresses due to surface peening is a plate with depth of h in the z direction, while coordinates origin coincides on the un-peened surface. The inherent symmetry of the shot peening process under the conditions of normal impact, results in equal residual stress distribution in x and y directions (Cao et al., 1995). Thus residual stress field is supposed to be independent of the x and y coordinates and its unknown variation in the z direction is assumed to be,

$$\sigma_{xx}(z) = \sigma_{yy}(z) = \sigma_{res}(z) \tag{1}$$

The residual stress field must satisfy equilibrium equations in absence of body forces and traction free boundary conditions at all boundaries (Timoshenko and Goodier, 1970). Furthermore Cartesian components of mean residual stress must be always zero (Mura, 1987). So zero mean residual stress for the introduced domain is reduced to,

$$\int_0^h \sigma_{res}(z)\, dz = 0 \tag{2}$$

In present approach, rather than direct determination of residual stress field from Airy stress function, the residual stress distribution is supposed to be,

$$\sigma_{res}(z) = \begin{cases} \sigma_{lnr}(z) & 0 < z < \tilde{h} \\ \sigma_{ind}(z) & \tilde{h} < z < h \end{cases} \tag{3}$$

where $\tilde{h}$ represents the depth of the shot peening affected zone and would be determined later based on the continuity condition of the reconstructed residual stress profile. Also $\sigma_{ind}(z)$ and $\sigma_{lnr}(z)$ are the induced stress and linear residual stress distribution defined within and outside of shot peening affected zone, respectively. In shot peening process, induced stresses are those encountered in a fully constrained component that create stretching and bending of the part to reach a balanced residual stress state (Van Luchene et al., 1995). In the proposed approach, both of the stretching and bending effects are regarded as the linear residual stresses.

The residual stresses measured in a thick component can be used to estimate induced stresses since the large thickness does not allow significant bending and stretching (Gariépy et al., 2011). Therefore induced stresses should have force balance across the domain of thick parts and its distribution is supposed to be,

$$\sigma_{ind}(z) = \frac{d}{dz}\left[z\,(h-z)F(z)\right] = (h-2z)F(z) + z\,(h-z)\frac{d}{dz}F(z) \tag{4}$$

Although the force equilibrium across the entire domain are obviously satisfied by the proposed form of the induced stresses as Eq. (4) but the moment static balance are not satisfied yet. To control the numerical behavior of the approximate solution and to ensure its existence and uniqueness, an arbitrary function of $F(z)$ is assumed to have the following asymptotic expansion,

$$F(z) = \sum_{m=1}^{M} \lambda_m f_m(z) \tag{5}$$

Where λ_m are unknown coefficients to be determined later based on the minimization of the deviation of measurements from its approximations and the smooth shape functions of $f_m(z)$ should exhibit the expected distribution of induced stresses in surface peening process. However it is well-known that the compressive residual stresses in the near-surface region have to be balanced by tensile residual stresses in the bulk of the specimen. Also the induced stress distribution is supposed to be concentrated in a relatively thin layer adjacent to the layer directly influenced by shot peening (Gariépy et al., 2011). Thus to impose the physical conditions of the shot peening induced stresses to shape functions, a family of shape functions is derived from a modulation function of $\Gamma(z)$ by translation and dilation as $f_m(z) = \Gamma\left(\zeta^m (z-h)\right)$. The Gaussian type modulation function of $\Gamma(z) = \exp(\gamma z)$ could be an appropriate choice that results in the expected distribution of induced stresses in the surface peening process. Therefore the explicit form of the shape functions are given by,

$$f_m(z) = \exp\left(\zeta^m \gamma (z-h)\right) \tag{6}$$

where the positive real constants of $1 < \zeta < \gamma$ govern the rate of convergence of the asymptotic solution and would be determined numerically based on the well-posedness of the solution considering relatively small condition number of the system. Accordingly, the final form of the induced stress would be expressed as a truncated series with M terms by,

$$\sigma_{ind}(z) = \sum_{m=1}^{M} \lambda_m \varphi_m(z) = \sum_{m=1}^{M} \lambda_m \left((h-2z) + z(h-z)\zeta^m \gamma\right) \exp\left(\zeta^m \gamma (z-h)\right) \tag{7}$$

The proposed form of shape functions will result in induced stress distribution exhibiting a compressive stress with the maximum value occurring at a distinct depth below the shot peened surface while concentrated in a relatively thin layer adjacent to the layer directly influenced by shot peening. Also the linear residual stress $\sigma_{lnr}(z) = c_0 + c_1 z$ is defined outside the shot peening affected zone and unknown coefficients of c_0, c_1 would be determined by satisfying the requirements of force and moment balance across the thickness of the surface peened plate as,

$$c_0 = \frac{2}{\tilde{h}^2}\int_{\tilde{h}}^{h}\sigma_{ind}(z)\left(3z - 2\tilde{h}\right)dz$$
$$c_1 = \frac{6}{\tilde{h}^3}\int_{\tilde{h}}^{h}\sigma_{ind}(z)\left(\tilde{h} - 2z\right)dz \tag{8}$$

Moreover in contrast to conventional reconstruction methods that assume the depth of the shot peening zone, it would be determined here based on the continuity condition of the reconstructed residual stress profile at the boundary of shot peening affected zone, i.e. $\sigma_{lnr}(\tilde{h}) = \sigma_{ind}\left(\tilde{h}\right)$.

Thus, the proposed approach results in a continuous reconstructed residual stress field while clearly satisfies stress equilibrium equations and traction free boundary conditions. Also the requirement of vanishing mean residual stress is guaranteed by satisfying the force and moment balance across the thickness of the surface peened plate.

2.2 Eigenstrains

The residual stress field in a continuum is not uniquely defined by the geometry, constitutive relations and boundary conditions as is the case for stress fields which arise solely from external loading (Timoshenko and Goodier, 1970). Instead, the residual stresses are dependent on the internal incompatibility within the continuum where the distribution of incompatible strain field in such a continuum is known as the eigenstrain distribution. Thus total strain tensor ε_{ij} can be expressed as the sum of elastic e_{ij} and eigenstrain terms ε_{ij}^* as $\varepsilon_{ij} = e_{ij} + \varepsilon_{ij}^*$ (Mura, 1987). Moreover kinematic analysis of continuous deformation requires that the total strain field to be compatible. Hence the total strain compatibility equation for equi-biaxial strains in the x and y directions, $\varepsilon_{xx}(z) = \varepsilon_{yy}(z) = \varepsilon(z)$, is reduced to (Timoshenko and Goodier, 1970),

$$\frac{d^2}{dz^2}\varepsilon(z) = \frac{d^2}{dz^2}e(z) + \frac{d^2}{dz^2}\varepsilon^*(z) = 0 \tag{9}$$

As a result, total strain should be a linear function of z to satisfy the strain compatibility equation. Motivated from fundamental assumptions of variational eigenstrain approach (Korsunsky, 2005); it is assumed that residual stress components are proportional to elastic strains and also the eigenstrain field outside the shot peening affected zone vanishes. Therefore the difference between the residual stress profile and linear residual stress is proportional to the eigenstrain field,

$$\varepsilon^*(z) = \frac{1}{\tilde{E}}\left(\sigma_{lnr}(z) - \sigma_{res}(z)\right) \tag{10}$$

where the elastic coefficient of $\tilde{E} = E/(1-\upsilon)$ may be readily found for biaxial stress state of shot peening process (Fathallah et al., 1998) and E and ν are Young's modulus and Poisson's ratio of the material, respectively. Furthermore in the case of extremely thick plates where the shot peening affected zone is extremely small compared to the thickness of the plate, linear residual stress approaches to zero and eigenstrain distribution is given by,

$$\varepsilon^{*}(z) = -\frac{1}{\tilde{E}}\sigma_{res}(z) \tag{11}$$

The direct proportionality of residual stress and permanent plastic strain in extremely thick plates subjected to uniform shot peening is in total agreement with variational eigenstrain approach (Korsunsky, 2005). In Section 3, the effects of component thickness as a geometric parameter influencing the residual stress state would be examined comprehensively. As a final point it should be emphasized that linearity of total strains is due to satisfy strain compatibility equations and is valid independent of plate bending theory assumptions.

2.3 Least squares analysis

Provided a limited set of experimental measurements consisting of the values of residual stress as T_n are measured at positions z_n, then evaluating the shape function of $\varphi_m(z)$ at each of n measurement positions, results in predicted values of $\varphi_{mn} = \varphi_m(z_n)$. However the unknown coefficients λ_m appeared in the asymptotic expansion of induced stress field cannot be directly evaluated by solving the over-determined set of linear algebraic equations of $\mathrm{T}_n = \sum_{m=1}^{M} \lambda_m \varphi_{mn}$. Since the number of truncated series M used to approximate the induced stress field should be less than the number of measurement points N, to attain a fast convergent series.

A standard alternative approach, as described earlier in (Faghidian, 2014, 2015), is to find the best values of coefficients λ_m that minimizes the deviation of the experimental measurements from its predicted values. Therefore the following norm of error should be minimized,

$$e = \left\| \mathrm{T}_n - \sum_{m=1}^{M} \lambda_m \varphi_{mn} \right\|^2 = \sum_{n=1}^{N} \left(\mathrm{T}_n - \sum_{m=1}^{M} \lambda_m \varphi_{mn} \right)^2 \tag{12}$$

It may be shown that unique values of coefficients $\mathbf{\Lambda} = \{\lambda_m\}$ can be determined by setting the gradient of error norm with respect to the coefficients equal to zero, $\partial e / \partial \lambda_m = 0$ (Faghidian, 2015). Therefore final form of the coefficients may be determined as,

$$\mathbf{\Lambda} = \left[\mathbf{\Phi}\mathbf{\Phi}^T \right]^{-1} \mathbf{\Phi}\mathbf{T} \tag{13}$$

where the matrix notation of $\mathbf{\Phi} = [\varphi_{mn}]$ and $\mathbf{T} = \{\mathrm{T}_n\}$ are introduced over the n=1...N measurement points and m=1...M the number of truncated series used to approximate the induced stress field.

Furthermore, a comprehensive discussion on the various mathematical aspects of the numerical reconstruction consisting of invertibility, uniqueness, well-posedness and convergence of asymptotic residual fields could be found in former works (Faghidian, 2014). It should be noted that the continuous dependence of the approximate solution on measured data is shown for shape functions derived from Gaussian type modulation function and the non-linear smooth shape functions are known to be numerically stabilized (Faghidian, 2014).

3 ANALYSIS OF EXPERIMENTAL RESULTS AND DISCUSSIONS

The presented framework of inverse determination of induced stress, residual stress and eigenstrain fields would be applied to three experimental data sets here. The experimental data examined here are measured in specimens with shallow shot peening affected zone where the depth of shot peening affected zone is $\tilde{h} \le h / 10$.

To reconstruct the residual fields, it is assumed that the magnitude of residual stresses is solely known at N measurement points within the shot peening affected zone, reported in Table 1 for every set of measurements. Also in each case studies, values of the parameters ζ and γ appeared in explicit form of shape functions, Eq. (6), are numerically determined based on the well-posedness of the system and are given in Table 1. Once the coefficients λ_m are evaluated, the induced and linear residual stresses and consequently the eigenstrain distribution over the entire domain would be determined. Moreover for each reconstructed residual field, number of the terms M utilized in truncated series form of residual field is tabulated in Table 1 to examine the rate of convergence issue.

As the first case study and to illustrate the strength of the present approach, the residual stress and eigenstrain profile determined by variational inverse eigenstrain analysis is examined. The aluminum alloy thick coupon with effective elastic modulus of $\tilde{E} = 103$ GPa has dimensions of $150 \times 50 \times 4$ mm and in tests is peened to full coverage using typical 0.5 mm shots (Levers and Prior, 1998). The residual stress and eigenstrain profile determined by Korsunsky (2005) utilizing variational inverse eigenstrain analysis are also shown in Fig. 1 and 2 and compared with the results of present approach, respectively. It is obviously inferred from Fig. 1 and Fig. 2 that the reconstructed residual fields by present approach are in excellent agreement with residual stress and eigenstrain profile of variational inverse eigenstrain analysis (Korsunsky, 2005) in the entire shot peening domain.

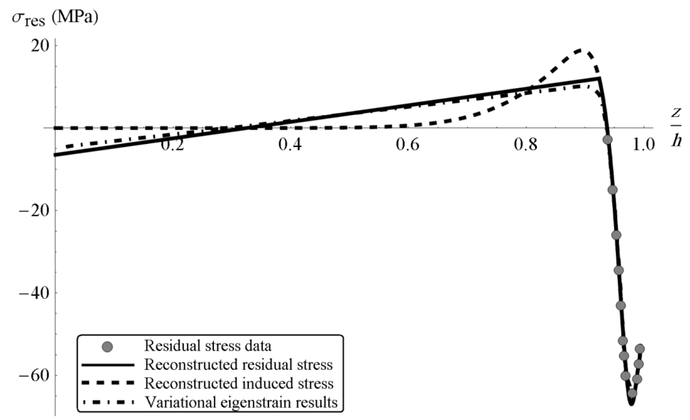


Figure 1: Residual stress data (Levers and Prior, 1998), reconstructed induced and residual stress distribution compared with variational inverse eigenstrain results (Korsunsky, 2005).

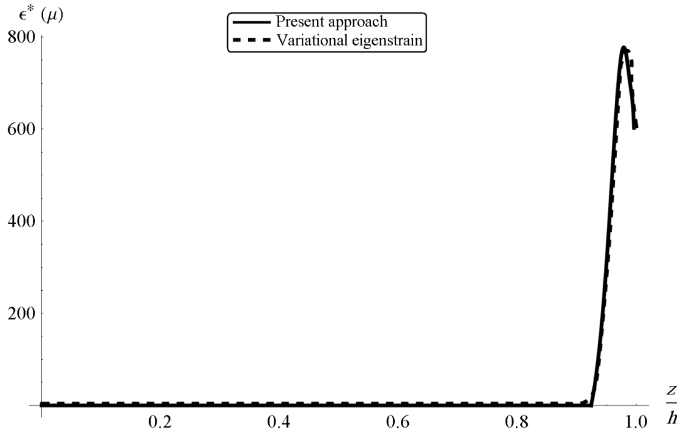
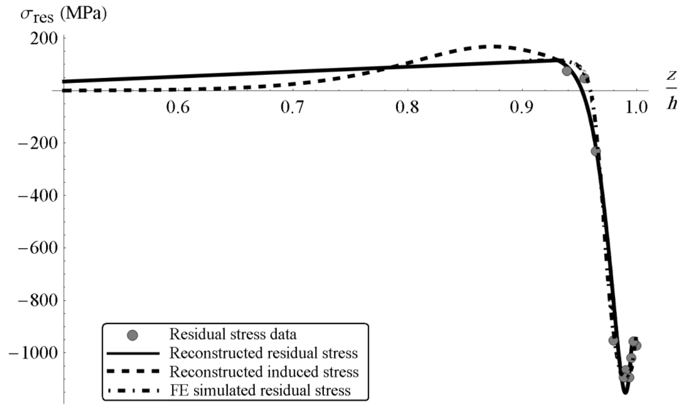


Figure 2: Reconstructed eigenstrain field compared with eigenstrain profile by variational inverse eigenstrain approach (Korsunsky, 2005).

Also to study the effects of plate thickness on the distribution of eigenstrain and residual stress fields, the X-ray diffraction measurements and FE simulated results of Zimmermann et al. (2009) is considered. The age hardened IN718 superalloy specimen with Young's modulus of E=200 GPa and Poisson's ratio of ν=0.32 is treated with an air blast machine. The test specimen featuring two sections of different thickness of 5 mm and 20 mm that is shot peened to 98 % coverage with a shot medium of type CCW31 with a measured mean diameter of 0.89 mm, a hardness of 550 HV, an impact angle of 80° and mean shot velocity of 23 m/s (Zimmermann et al., 2009). The residual stress profiles simulated by FE analysis using an elasto-viscoplastic material model with combined isotropic and kinematic hardening (Zimmermann et al., 2009) are also shown in Fig. 3(a) and 3(b) for thickness of 5 mm and 20 mm, respectively. The reconstructed induced stress, residual stress and eigenstrain distribution by present approach are also exhibited together with X-ray measurements and simulated residual stress depth profiles for different thicknesses in Fig. 3 and Fig. 4, respectively.

(a)



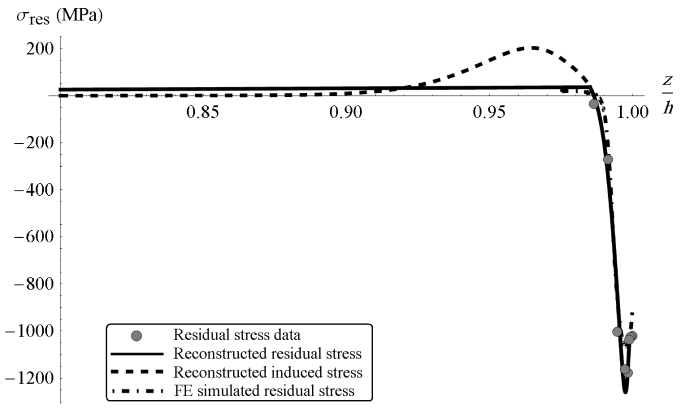
(b)


Figure 3: Reconstructed induced and residual stress distribution compared with X-ray measurements (Zimmermann et al., 2009) and simulated residual stress profile (Zimmermann et al., 2009) for thickness of (a) 5 mm, (b) 20 mm.

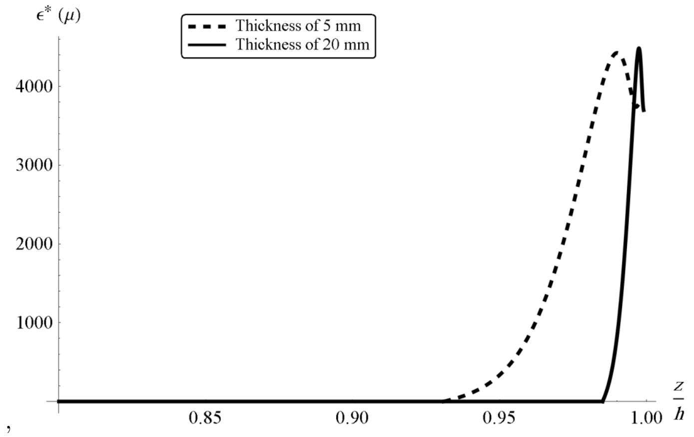


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Figure 4: Reconstructed eigenstrain depth profile for X-ray measurements of Zimmermann et al. (2009) with thickness of 5 mm and 20 mm.

It is clearly inferred from Fig. 3 that decreasing the component thickness does not affect the maximum magnitude of compressive residual stresses in shot peening affected region but leads to higher tensile residual stresses in greater depths. Also the difference in magnitude of tensile stresses and gradient of linear residual stress can be attributed to the different thicknesses available to balance the compressive residual stresses. It should be noted that increasing the thickness of surface peened plates would result in lower magnitudes of linear residual stress which is sometimes less than the resolution of measurement methods; as a result the linear residual stress is not constant through the thickness and still varies linearly even if with small gradient.

Furthermore it is obviously deduced from Fig. 4 that the maximum magnitude of eigenstrain field is approximately the same and identically independent of the plate thickness but the eigenstrain field is distributed over the larger zone for smaller thickness. The results are in complete agreement with FE simulated plastic deformation which the maximum magnitudes are always identically independent of FE model thickness (Zimmermann et al., 2009).

	ζ	γ	N	M
Korsunsky (2005) data	2.1	9.5	12	6
Zimmermann et al. (2009)- 5 mm thickness	2.9	8	10	6
Zimmermann et al. (2009)- 20 mm thickness	3	10	8	6

Table 1: Values of shape function parameters, number of measurement points and terms used for residual stress reconstruction.

However it worth to note that, although the induced stresses have the same profile as the residual stresses in shot peening affected zone but induced stresses cannot represent the residual field in deeper regions since not only they do not satisfy the moment balance across the plate thickness but also would result in negative eigenstrain distribution near to shot peening affected zone which is not acceptable (Korsunsky, 2009). Thus the compressive residual stresses generated by surface peening procedure in the shot peening affected region could be only balanced by rather low residual stresses varying linearly over the whole cross-section as opposed to relatively high tensile stresses in a near-surface layer.

In reconstruction of residual fields in surface peened plates, a full range of continuum mechanics requirements are satisfied. Furthermore an optimal agreement between experimental measurements and model predictions is attained in the least squares sense. The eigenstrain field is also inversely determined by satisfying strain compatibility equations while in conventional reconstruction methods generally a direct approach would be utilized to determine the residual stresses from an assumed eigenstrain distribution.

The open mathematical problem concerns the development of present approach for appropriate choice of unknown parameters ζ,γ of shape functions $f_m(z)$. Although a variety of examples exists for numerical selection of such variable parameters in stress function approach (Faghidian, 2014, 2015; Farrahi et al. 2009a, 2009b; Faghidian et al., 2012) based on the well-posedness of the reconstructed solution considering relatively small condition number of the system of linear equations (Allaire and Kaber, 2008) appearing in the equation of $\partial e / \partial \lambda_m = 2\mathbf{\Phi}\left(\mathbf{\Phi}^T \mathbf{\Lambda} - \mathbf{T}\right) = 0$ and the fast convergence rate of the solution; but no theoretical results are available at present time.

Nonetheless numerical determination of appropriate values for shape function parameters results in a truncated series with six terms that has a fast convergence. It should be noted that, the convergence criterion for the asymptotic series would be defined as the norm of the discrepancy of successive approximations is of the order of the noise in data. Also a good estimate for the measurement error could be the unbiased estimator for variance of the measured data (Faghidian, 2015).

4 CONCLUSIONS

An examination of the eigenstrain field, induced and residual stresses in surface peened specimens with shallow shot peening affected zone, has revealed the points outlined below.

1. The well-rehearsed advantage of the proposed approach is that it not only minimizes the deviation of the residual measurements from its predicted values but also will result in an inverse solution satisfying a full range of continuum mechanics requirements.
2. In contrast to conventional reconstruction methods, the eigenstrain field is analytically determined by satisfying the strain compatibility equation.
3. The residual stress distribution not only depends on the compressive residual stresses in the surface region but also depends on the component thickness as an influencing geometric parameter.
4. The plate thickness has no influence on the maximum magnitude of eigenstrain profile within the shot peening affected region but would effectively modify the magnitude and gradient of linear residual stresses present in deeper regions.
5. Although lower linear residual stress would be expected by increasing the thickness of surface peened plate but the residual stress is not constant in the core regions and still varies linearly even if with small gradient.

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