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Combination of Modified Yld2000-2d and Yld2000-2d in Anisotropic Pressure Dependent Sheet Metals

Abstract

In the current research to model anisotropic asymmetric sheet metals a new non-AFR criterion is presented. In the new model, Modified Yld2000-2d proposed by Lou et al. (2013) is considered as yield function and Yld2000-2d proposed by Barlat et al. (2003) is considered as plastic potential function. To calíbrate the presented criterion, the yield function which is a pressure dependent criterion requiers ten directional yield stresses such as uniaxial tensile stresses in three directions of 0°, 45°, 90°, uniaxial compressive yield stresses in six directions of 0°, 15°, 30°, 45°, 75°, 90° from the rolling direction along with biaxial yield stress. Moreover, the plastic potential function which is a pressure indepedent criterion needs eight experimental data points such as tensile *R*-values in seven directions of 0°, 15°, 30°, 45°, 60°, 75°, 90° from the rolling direction and also biaxial tensile *R*-value. Finally with comparing the obtained results with experimental data points, it is shown that the presented non-AFR criterion predicts compressive yield stress, biaxial tensile yield stress and *R*-values more accurately than Modified Yld2000-2d and it would be considered as a new criterion for anisotropic asymmetric metals.

Keywords

Yld2000-2d, Modified Yld2000-2d, Anisotropic asymmetric yield functions, Non-associated flow rule, Downhill Simplex Method.

Farzad Moayyedian [a]
Mehran Kadkhodayan [b]

[a] Ph. D. Student of Solid Mechanical Engineering, Ferdowsi University of Mashhad, Iran
farzad.moayyedian@gmail.com
[b] Professor of Solid Mechanical Engineering, Ferdowsi University of Mashhad, Iran
kadkhoda@um.ac.ir

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1 INTRODUCTION

To model the behavior of anisotropic metals more precisely, their pressure dependency is considered in the recent new studies. Spitz and Richmond (1984) by the aid of experimental data of iron-based materials and aluminum showed that there is no need that the pressure dependency of yielding to be associated with irreversible plastic dilatancy. Liu et al. (1997) considered tensile and compressive strengths far apart and presented a new criterion based on the criterion presented by Hill and Drucker-Prager. Barlat et al. (1997) proposed a generalized yield description to account for the behavior of the strengthened aluminum alloy sheets. It was subsequently shown that this yield function was suitable for the description of the plastic behavior of any aluminum alloy sheet. Barlat et al. (2003) used two linear transformations on the Cauchy stress tensor to propose a new plane stress yield function for aluminum alloy sheets to consider the anisotropy effects. Stoughton and Yoon (2004) proposed a non-AFR based on a pressure sensitive yield description with isotropic hardening to account the strength differential effect (SDE) that was consistent with the Spitzig and Richmond (1984) results. Lee et al. (2008) developed a continuum plasticity model to consider the unusual plastic behavior of magnesium alloy in finite element analysis. A hardening law based on two-surface model was further extended to consider the general stress–strain response of sheet metals including the Bauschinger effect, transient behavior and unusual asymmetry. In terms of anisotropy and asymmetry of the initial yield stress, the Drucker–Prager's pressure dependent yield surface was modified to include the anisotropy of magnesium alloy. Cvitanić et al. (2008) developed a finite element formulation based on non-associated plasticity. In the constitutive formulation, isotropic hardening was assumed and an equation for the hardening parameter consistent with the principle of plastic work equivalence was introduced. The yield function and plastic potential function were considered as two different functions. Algorithmic formulations of constitutive models that utilized associated or non-AFR were derived by application of implicit return mapping procedure. Aretz (2009) presented the issue of the yield function's convexity in the existence of hydrostatic pressure for sensitive yield conditions. Stoughton and Yoon (2009) proposed a model based on a non-AFR and explicitly integrated it into the yield criterion. Their results had no effects on the accuracy of the plastic strain components defined by the gradient of a separate plastic potential function. Hu and Wang (2009) proposed a new theory in which a corresponding constitutive model can be constructed and characterized experimentally via two steps. The constitutive model involved two functions, yield function and plastic potential. A relationship between two functions was suggested. Drucker–Prager yield function was employed to express the yielding behavior of material and a differently experimental characterization of the model was created as the corresponding plastic potential to describe the feature of plastic flow of material. The constitutive model could also well predict stress–strain relations with different pressures loaded on the material. They showed that the feature of plastic flow was not that sensitive to the pressure loaded on the material when the yielding stress was. Huh et al. (2010) evaluated the accuracy of accepted anisotropic yield criteria contain Hill48, Yld89, Yld91, Yld96, Yld2000-2d, BBC2000 and Yld2000-18p rooted in the root-mean square error (RMSE) of the yield stresses and the R-values. They concluded that Yld2000-18 yield criterion was the most proper one to precisely describe the yield tress and R-value directionalities of sheet metals. Taherizadeh et al. (2011) extended a generalized finite element formulation of stress integration method for non-quadratic yield and potential functions with combined non-linear hardening with non-AFR. Park and Chung (2012) developed a novel formulation under the structure of mixed isotropic-kinematic hardening rule which led to the symmetric stiffness modulus for the non-AFR. Cleja-Ţigoiu and Lancu (2013) introduced the rate elastic-plastic model with a plastic spin in the case of an in-plane rotation of the orthotropy direction and a plane stress condition, respectively. In the plane stress case, the equation for the strain rate in the normal direction was first derived and subsequently the modified expression for the plastic multiplier associated with an in-plane rate of the deformation became available. Numerical simulations for the homogeneous deformation process on the sheets and comparisons with experimental data made possible a selection among the plastic spins introduced in their research, aiming at obtaining a good agreement with the experiments performed for an in-plane stress state. Lou et al. (2013) introduced a method to extend symmetric yield functions to concern the strength differential (SD) effect in sheet metals. Adding a weighted pressure term for anisotropic materials the SD effect was coupled with symmetric yield functions. This method was applied to the symmetric Yld2000-2d and this yield function was modified to describe the anisotropic and a symmetric yielding of two aluminum alloys with small and strong SD effects. Safaei et al. (2013) proposed a non-associated plane stress anisotropic constitutive model with combined isotropic-kinematic hardening. The quadratic Hill48 and non-quadratic Yld2000-2d yield criteria were used with the non-AFR model to consider anisotropic behavior. Safaei et al. (2014) introduced anisotropy evolution in terms of both distortional hardening and variation of Lankford coefficients. A non-AFR based on Yld2000-2d anisotropic yield model was used in which separate yield function and plastic potential were considered which could provide excellent accuracy and flexibility for the model. Safaei et al. (2014) showed that by scaling the plastic potential function, the equality of equivalent plastic strain and compliance factor can be reserved. The effect of scaling of the non-AFR based on Barlat et al.'s (2003) anisotropic model (called Yld2000-2d) was comprehensively studied with FE simulation of tensile loading under uniaxial tensions along with different orientations as well as balanced biaxial stress condition. A fully implicit return-mapping scheme was introduced for stress integration of the constitutive model in a User-defined MATerial subroutine (UMAT). The results proved that the proposed simplified technique was a reliable alternative for the full expression. Yoon et al. (2014) proposed a general asymmetric yield function with dependence on the stress invariants for pressure sensitive metals. The pressure sensitivity of the proposed yield function was consistent with the experimental result of Spitzig and Richmond (1984) for steel and aluminum alloys while the asymmetry of the third invariant was preserved to model strength differential (SD) effect of pressure insensitive materials. The proposed yield function was transformed in the space of the stress triaxaility, the von Mises stress and the normalized invariant to theoretically investigate the possible reason of the SD effect. The yield function reasonably modeled the evolution of yield surfaces for a zirconium clock-rolled plate during in-plane and through thickness compression. The yield function was also applied to describe the orthotropic behavior of a face-centered cubic metal of AA2008-T4 and two hexagonal close-packed metals of high-purity a-titanium and AZ31 magnesium alloy.

In the current research the modified Yld2000-2d and Yld2000-2d are used as yield and plastic potential functions in new criterion called Modified Yld2000-2d II. The yield and the plastic potential functions are calibrated with ten and eight experimental results, respectively. Finally it is shown that the Modified Yld2000-2d II predicts σ_θ^C, σ_b^T, R_θ^T and R_b^T more accurately than new Modified Yld2000-2d presented by Lou et al. (2013) for anisotropic pressure dependent metals Al2008-T4 (a BCC material) and Al2090-T3 (a FCC material).

2 MODIFIED Yld2000-2d II IN PLANE STRESS PROBLEMS

In the current research a new non-AFR is introduced (Modified Yld2000-2d II) in which the Modified Yld2000-2d proposed by Lou et al. (2013) and Yld2000-2d proposed by Barlat et al. (2003) are selected as yield and plastic potential functions, respectively. In the proposed model the pressure dependency of yield function and pressure independency of plastic potential function are considered accurately. Moreover, the number of experimental data points for calibrating the proposed criterion are increased from ten in Modified Yld2000-2d to eighteen. In the following, calibrating of the Modified Yld2000-2d II is explained in detail.

To consider the anisotropic effects in yield stress function, two linear transformation matrices (L'_{ij} and L''_{ij}) in stress tensor are introduced. Then the first modified deviatoric stress tensor X'_{ij} can be presented by Barlat et al. (2003) as bellow:

$$\begin{Bmatrix} X'_{xx} \\ X'_{yy} \\ X'_{xy} \end{Bmatrix} = \begin{bmatrix} L'_{11} & L'_{12} & 0 \\ L'_{21} & L'_{22} & 0 \\ 0 & 0 & L'_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} \tag{1}$$

in which σ_{ij} is the stress tensor and L'_{ij} can be expressed with three anisotropic parameters α_1, α_2 and α_7 as following:

$$\begin{Bmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{66} \end{Bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{Bmatrix} \tag{2}$$

In this case the first modified principal deviatoric stresses X'_1 and X'_2 can be shown as:

$$X'_1, X'_2 = \frac{1}{2}\left(X'_{xx} + X'_{yy} \pm \sqrt{\left(X'_{xx} - X'_{yy} \right)^2 + 4X'^2_{xy}} \right) \tag{3}$$

and the second modified deviatoric stress tensor X''_{ij} can be introduced as:

$$\begin{Bmatrix} X''_{xx} \\ X''_{yy} \\ X''_{xy} \end{Bmatrix} = \begin{bmatrix} L''_{11} & L''_{12} & 0 \\ L''_{21} & L''_{22} & 0 \\ 0 & 0 & L''_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} \tag{4}$$

in which L''_{ij} can be shown with five anisotropic parameters of α_3, α_4, α_5, α_6 and α_8 as following:

$$\begin{Bmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{66} \end{Bmatrix} = \frac{1}{9}\begin{bmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix}\begin{Bmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{Bmatrix} \tag{5}$$

In this case the second modified principal deviatoric stresses X''_1 and X''_2 can be expressed as:

$$X''_1, X''_2 = \frac{1}{2}\left(X''_{xx} + X''_{yy} \pm \sqrt{\left(X''_{xx} - X''_{yy} \right)^2 + 4X''^2_{xy}} \right) \tag{6}$$

Using these principal deviatoric stresses, the Modified Yld2000-2d can be obtained by Lou et al. (2013) equation as:

$$F = h_x \sigma_{xx} + h_y \sigma_{yy} + \left(\frac{\left| X'_1 - X'_2 \right|^a + \left| 2X''_2 + X''_1 \right|^a + \left| 2X''_1 + X''_2 \right|^a}{2} \right)^{\frac{1}{a}} = \sigma\left(\bar{\varepsilon}^p \right) \tag{7}$$

in which h_x and h_y are the parameters for considering the pressure dependency of the anisotropic criterion which is an anisotropic asymmetric (pressure dependent) one. In this criterion $a = 6$ for BCC and $a = 8$ for FCC materials are the best choices and $\sigma\left(\bar{\varepsilon}^p \right)$ defines the isotropic hardening behavior of materials considered by Lou et al. (2013).

Similarly for plastic potential function it can be shown that Y'_2 (the first modified deviatoric stress tensor) can be expressed with a linear transformation M'_{ij} such as:

$$\begin{Bmatrix} Y'_{xx} \\ Y'_{yy} \\ Y'_{xy} \end{Bmatrix} = \begin{bmatrix} M'_{11} & M'_{12} & 0 \\ M'_{21} & M'_{22} & 0 \\ 0 & 0 & M'_{66} \end{bmatrix}\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} \tag{8}$$

where M'_{ij} can be presented with three anisotropic parameters of β_1, β_2 and β_7 as following:

$$\begin{Bmatrix} M'_{11} \\ M'_{12} \\ M'_{21} \\ M'_{22} \\ M'_{66} \end{Bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_7 \end{Bmatrix} \tag{9}$$

where the first modified principal deviatoric stresses Y_1' and Y_2' are:

$$Y_1', Y_2' = \frac{1}{2}\left(Y'_{xx} + Y'_{yy} \pm \sqrt{\left(Y'_{xx} - Y'_{yy}\right)^2 + 4Y'^2_{xy}}\right) \tag{10}$$

and Y''_{ij} (the second modified deviatoric stress tensor) can be presented with another linear transformation M''_{ij} as:

$$\begin{Bmatrix} Y''_{xx} \\ Y''_{yy} \\ Y''_{xy} \end{Bmatrix} = \begin{bmatrix} M''_{11} & M''_{12} & 0 \\ M''_{21} & M''_{22} & 0 \\ 0 & 0 & M''_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} \tag{11}$$

in which M''_{ij} are presented with five anisotropic parameters of β_3, β_4, β_5, β_6 and β_8 as:

$$\begin{Bmatrix} M''_{11} \\ M''_{12} \\ M''_{21} \\ M''_{22} \\ M''_{66} \end{Bmatrix} = \frac{1}{9}\begin{bmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix} \begin{Bmatrix} \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_8 \end{Bmatrix} \tag{12}$$

where the modified principal deviatoric stresses Y_1'' and Y_2'' are:

$$Y_1'', Y_2'' = \frac{1}{2}\left(Y''_{xx} + Y''_{yy} \pm \sqrt{\left(Y''_{xx} - Y''_{yy}\right)^2 + 4Y''^2_{xy}}\right) \tag{13}$$

Finally the Yld2000-2d for anisotropic pressure independent materials can be considered as:

$$G = \left(\frac{\left|Y_1' - Y_2'\right|^a + \left|2Y_2'' + Y_1''\right|^a + \left|2Y_1'' + Y_2''\right|^a}{2}\right)^{\frac{1}{a}} \tag{14}$$

In this case $a = 6$ for BCC and $a = 8$ for FCC materials are the best choices as Modified Yld2000-2d mentioned by Barlat et al. (2003).

It is noted that Eqs. (8) - (13) for plastic potential function are derived similary to analogy in Eqs. (1) - (6) for yield function. The yield function in Eq. (7) is pressure dependent but the plastic potential function in Eq. (14) is pressure independent. To calibrate the Yld2000-2d as plastic function the first differentiation of this criterion with respect to σ_{ij} is derived as following:

$$\begin{cases}\dfrac{\partial G}{\partial \sigma_{xx}}=\dfrac{G^{1-a}}{2}\times[\left(2\beta_1+\beta_2\right)\times\dfrac{\left(2\beta_1+\beta_2\right)\sigma_{xx}-\left(\beta_1+2\beta_2\right)\sigma_{yy}}{18K_2'}\left(2K_2'\right)^{a-1}+\\ \left(\dfrac{3}{2}\left(M_{11}''+M_{21}''\right)-\left(M_{11}''-M_{21}''\right)\times\dfrac{\left(M_{11}''-M_{21}''\right)\sigma_{xx}+\left(M_{12}''-M_{22}''\right)\sigma_{yy}}{4K_2''}\right)\left|3K_1''-K_2''\right|^{a-1}+\\ \left(\dfrac{3}{2}\left(M_{11}''+M_{21}''\right)+\left(M_{11}''-M_{21}''\right)\times\dfrac{\left(M_{11}''-M_{21}''\right)\sigma_{xx}+\left(M_{12}''-M_{22}''\right)\sigma_{yy}}{4K_2''}\right)\left|3K_1''+K_2''\right|^{a-1}]\\ \dfrac{\partial G}{\partial \sigma_{yy}}=\dfrac{G^{1-a}}{2}\times[-\left(\beta_1+2\beta_2\right)\times\dfrac{\left(2\beta_1+\beta_2\right)\sigma_{xx}-\left(\beta_1+2\beta_2\right)\sigma_{yy}}{18K_2'}\left(2K_2'\right)^{a-1}+\\ \left(\dfrac{3}{2}\left(M_{12}''+M_{22}''\right)-\left(M_{12}''-M_{22}''\right)\times\dfrac{\left(M_{11}''-M_{21}''\right)\sigma_{xx}+\left(M_{12}''-M_{22}''\right)\sigma_{yy}}{4K_2''}\right)\left|3K_1''-K_2''\right|^{a-1}+\\ \left(\dfrac{3}{2}\left(M_{12}''+M_{22}''\right)+\left(M_{12}''-M_{22}''\right)\times\dfrac{\left(M_{11}''-M_{21}''\right)\sigma_{xx}+\left(M_{12}''-M_{22}''\right)\sigma_{yy}}{4K_2''}\right)\left|3K_1''+K_2''\right|^{a-1}]\\ \dfrac{\partial G}{\partial \tau_{xy}}=\dfrac{G^{1-a}}{2}\times\left(2\dfrac{\beta_7^2}{K_2'}\left(2K_2'\right)^{a-1}-\dfrac{\left(M_{66}''\right)^2}{K_2''}\left|3K_1''-K_2''\right|^{a-1}+\dfrac{\left(M_{66}''\right)^2}{K_2''}\left|3K_1''+K_2''\right|^{a-1}\right)\tau_{xy}\end{cases} \tag{15}$$

where

$$\begin{cases}K_1'=\dfrac{\left(2\beta_1-\beta_2\right)\sigma_{xx}-\left(\beta_1-2\beta_2\right)\sigma_{yy}}{6}\\ K_2'=\sqrt{\left(\dfrac{\left(2\beta_1+\beta_2\right)\sigma_{xx}-\left(\beta_1+2\beta_2\right)\sigma_{yy}}{6}\right)^2+\left(\beta_7\tau_{xy}\right)^2}\\ K_1''=\dfrac{\left(M_{11}''+M_{21}''\right)\sigma_{xx}+\left(M_{12}''+M_{22}''\right)\sigma_{yy}}{2}\\ K_2''=\sqrt{\left(\dfrac{\left(M_{11}''-M_{21}''\right)\sigma_{xx}+\left(M_{12}''-M_{22}''\right)\sigma_{yy}}{2}\right)^2+\left(M_{66}''\tau_{xy}\right)^2}\end{cases} \tag{16}$$

3 CALIBRATION OF MODIFIED Yld2000-2d II IN PLANE STRESS PROBLEMS

In this part the yield function of Modified Yld2000-2d II which is an asymmetric function (pressure dependent criterion) is calibrated with ten stress experimental data points such as uniaxial tensile yield stresses $\left(\sigma_\theta^T\right)$ in 0°, 45°, 90°, uniaxial compressive yield stresses $\left(\sigma_\theta^C\right)$ in 0°, 15°, 30°, 45°, 75°, 90° from rolling direction along with biaxial yield stress, $\left(\sigma_b^T\right)$. On the other hand, the plastic potential function of Modified Yld2000-2d II which is a symmetric function (pressure independent criterion) is calibrated with eight experimental data such as *R*-values $\left(R_\theta^T = \dfrac{d\varepsilon_{yy}^p}{d\varepsilon_{zz}^p}\right)$ in 0°, 15°, 30°, 45°, 60°, 75°, 90° from the rolling direction and also biaxial tensile *R*-value $\left(R_b^T = \dfrac{d\varepsilon_{yy}^p}{d\varepsilon_{xx}^p}\right)$. Since the yield function is pressure dependent, the experimental tests should be employed in tensile and compressive directions. In tensile test (in θ angle from rolling direction) the stress tensor will be:

$$\begin{cases} \sigma_{xx} = \sigma_\theta^T cos^2\theta \\ \sigma_{yy} = \sigma_\theta^T sin^2\theta \\ \tau_{xy} = \sigma_\theta^T sin\theta cos\theta \end{cases} \tag{17}$$

and similarly in compression test it is found that:

$$\begin{cases} \sigma_{xx} = -\sigma_\theta^C cos^2\theta, \\ \sigma_{yy} = -\sigma_\theta^C sin^2\theta, \\ \tau_{xy} = -\sigma_\theta^C sin\theta cos\theta. \end{cases} \tag{18}$$

For biaxial test the state of stress is as below:

$$\begin{cases} \sigma_{xx} = \sigma_b^T, \\ \sigma_{yy} = \sigma_b^T, \\ \tau_{xy} = 0. \end{cases} \tag{19}$$

Substituting these values in Eq. (7) the tensile and compressive yield stresses in different orientations from the rolling direction and also biaxial yield stress are obtained as following:

$$\begin{cases} \sigma_\theta^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{h_x cos^2\theta + h_y sin^2\theta + \left(\dfrac{\left|2K'_{1p}\right|^a + \left|3K''_{1p} - K''_{2p}\right|^a + \left|3K''_{1p} + K'_{1p}\right|^a}{2}\right)^{\frac{1}{a}}} \\ \sigma_\theta^C = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{-h_x cos^2\theta - h_y sin^2\theta + \left(\dfrac{\left|2K'_{1p}\right|^a + \left|-3K''_{1p} + K''_{2p}\right|^a + \left|-3K'_{1p} - K''_{2p}\right|^a}{2}\right)^{\frac{1}{a}}} \\ \sigma_b^T = \dfrac{\sigma\left(\bar{\varepsilon}^p\right)}{h_x + h_y + \left(\dfrac{\left|\dfrac{\alpha_1 - \alpha_2}{3}\right|^a + \left|L''_{11} + 2L''_{21} + L''_{12} + 2L''_{22}\right|^a + \left|2L''_{11} + L''_{21} + 2L''_{12} + L''_{22}\right|^a}{2}\right)^{\frac{1}{a}}} \end{cases} \tag{20}$$

in which,

$$\begin{cases} K'_{1p} = \dfrac{\left(2\alpha_1 - \alpha_2\right)cos^2\theta - \left(\alpha_1 - 2\alpha_2\right)sin^2\theta}{6} \\ K'_{2p} = \sqrt{\left(\dfrac{\left(2\alpha_1 + \alpha_2\right)cos^2\theta - \left(\alpha_1 + 2\alpha_2\right)sin^2\theta}{6}\right)^2 + \left(\alpha_7 sin\theta cos\theta\right)^2} \\ K''_{1p} = \dfrac{\left(L''_{11} + L''_{21}\right)cos^2\theta + \left(L''_{12} + L''_{22}\right)sin^2\theta}{2} \\ K''_{2p} = \sqrt{\left(\dfrac{\left(L''_{11} - L''_{21}\right)cos^2\theta + \left(L''_{12} - L''_{22}\right)sin^2\theta}{2}\right)^2 + \left(L''_{66} sin\theta cos\theta\right)^2} \end{cases} \tag{21}$$

Using Eqs. (20) and (21) along with stress components in Eqs. (17) to (19), the Modified Yld2000-2d as a yield function can be calibrated.

In the current research the non-AFR is employed and the Yld2000-2d as a pressure independent criterion is used for plastic potential function. Using non-AFR rule it is found that:

$$\begin{cases} d\varepsilon_{xx}^{p} = d\lambda \dfrac{\partial G}{\partial \sigma_{xx}} \\ d\varepsilon_{yy}^{p} = d\lambda \dfrac{\partial G}{\partial \sigma_{yy}} \\ d\varepsilon_{xy}^{p} = d\lambda \dfrac{\partial G}{\partial \tau_{xy}} \end{cases} \tag{22}$$

With considering the new presented plastic potential function in Eq. (14) which is pressure independent the assumption of incompressibility can be stated as bellow:

$$d\varepsilon_{zz}^{p} = -d\varepsilon_{xx}^{p} - d\varepsilon_{yy}^{p} \tag{23}$$

Moreover, from the definition of *R*-values it can be shown that:

$$\begin{cases} R_{\theta}^{T} = \dfrac{d\varepsilon_{yy}^{p}}{d\varepsilon_{zz}^{p}} = -\dfrac{\dfrac{\partial G}{\partial \sigma_{xx}} sin^{2}\theta + \dfrac{\partial G}{\partial \sigma_{yy}} cos^{2}\theta - \dfrac{\partial G}{\partial \tau_{xy}} sin\theta cos\theta}{\dfrac{\partial G}{\partial \sigma_{xx}} + \dfrac{\partial G}{\partial \sigma_{yy}}} \\ R_{b}^{T} = \dfrac{d\varepsilon_{yy}^{p}}{d\varepsilon_{xx}^{p}} = \dfrac{\dfrac{\partial G}{\partial \sigma_{yy}}}{\dfrac{\partial G}{\partial \sigma_{xx}}} \end{cases} \tag{24}$$

Using the stress components in Eqs. (17) to (19) and also $\dfrac{\partial G}{\partial \sigma_{ij}}$ obtained from Eqs. (15) and (16) the Yld2000-2d as a plastic potential function can be calibrated.

4 PARAMETER EVALUATION AND ROOT MEAN SQUARE ERRORS OF THE YIELD STRESSES AND *R*-VALUES

Ten material constants denoted as $\alpha_i \left(i = 1,8\right)$, h_x and h_y for the yield function and also eight parameters of $\beta_i \left(i = 1,8\right)$ for the plastic potential function can be computed from eighteen experimental data points such as as σ_0^T, σ_{45}^T, σ_{90}^T, σ_b^T, σ_0^C, σ_{15}^C, σ_{45}^C, σ_{60}^C, σ_{75}^C, σ_{90}^C, R_0^T, R_{15}^T, R_{30}^T, R_{45}^T, R_{60}^T, R_{75}^T, R_{90}^T and R_b^T for Modified Yld2000-2d II. These experimental data points are utilized to set up two error functions, i.e. the first for the yield function $\left(E_1\right)$ and the second for the plastic function $\left(E_2\right)$ of Modified Yld2000-2d II as following:

$$E_1=\left[\frac{\left(\sigma_0^T\right)_{pred.}}{\left(\sigma_0^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(\sigma_{30}^T\right)_{pred.}}{\left(\sigma_{30}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(\sigma_{45}^T\right)_{pred.}}{\left(\sigma_{45}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(\sigma_{90}^T\right)_{pred.}}{\left(\sigma_{90}^T\right)_{exp.}}-1\right]^2+$$
$$\left[\frac{\left(\sigma_b^T\right)_{pred.}}{\left(\sigma_b^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(\sigma_0^C\right)_{pred.}}{\left(\sigma_0^C\right)_{exp.}}-1\right]^2+\left[\frac{\left(\sigma_{15}^C\right)_{pred.}}{\left(\sigma_{15}^C\right)_{exp.}}-1\right]^2+\left[\frac{\left(\sigma_{45}^C\right)_{pred.}}{\left(\sigma_{45}^C\right)_{exp.}}-1\right]^2+\left[\frac{\left(\sigma_{60}^C\right)_{pred.}}{\left(\sigma_{60}^C\right)_{exp.}}-1\right]^2+ \quad (25)$$
$$\left[\frac{\left(\sigma_{90}^C\right)_{pred.}}{\left(\sigma_{90}^C\right)_{exp.}}-1\right]^2=0$$

and

$$E_2=\left[\frac{\left(R_0^T\right)_{pred.}}{\left(R_0^T\right)_{exp.}}-1\right]+\left[\frac{\left(R_{15}^T\right)_{pred.}}{\left(R_{15}^T\right)_{exp.}}-1\right]+\left[\frac{\left(R_{30}^T\right)_{pred.}}{\left(R_{30}^T\right)_{exp.}}-1\right]+\left[\frac{\left(R_{45}^T\right)_{pred.}}{\left(R_{45}^T\right)_{exp.}}-1\right]^2$$
$$+\left[\frac{\left(R_{60}^T\right)_{pred.}}{\left(R_{60}^T\right)_{exp.}}-1\right]+\left[\frac{\left(R_{75}^T\right)_{pred.}}{\left(R_{75}^T\right)_{exp.}}-1\right]+\left[\frac{\left(R_{90}^T\right)_{pred.}}{\left(R_{90}^T\right)_{exp.}}-1\right]^2+\left[\frac{\left(R_b^T\right)_{pred.}}{\left(R_b^T\right)_{exp.}}-1\right]^2 \quad (26)$$

These error functions can be minimized by the Downhill Simplex method to identify the material parameters which were more explained by Hu et al. (2010) and Barlat et al. (2013).
The root-mean square errors (RMSEs) of the tensile $\left(E_\sigma^T\right)$ and compressive $\left(E_\sigma^C\right)$ yield stresses, the tensile biaxial stress $\left(E_\sigma^{Tb}\right)$, the tensile *R*-values $\left(E_R^T\right)$ and the tensile biaxial *R*-value $\left(E_\sigma^{Tb}\right)$ can be computed as follow:

$$E_\sigma^T=\frac{1}{7}\left(\begin{array}{l}\left[\frac{\left(\sigma_0^T\right)_{exp.}-\left(\sigma_0^T\right)_{pred.}}{\left(\sigma_0^T\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{15}^T\right)_{exp.}-\left(\sigma_{15}^T\right)_{pred.}}{\left(\sigma_{15}^T\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{30}^T\right)_{exp.}-\left(\sigma_{30}^T\right)_{pred.}}{\left(\sigma_{30}^T\right)_{exp.}}\right]^2+\\ \left[\frac{\left(\sigma_{45}^T\right)_{exp.}-\left(\sigma_{45}^T\right)_{pred.}}{\left(\sigma_{45}^T\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{60}^T\right)_{exp.}-\left(\sigma_{60}^T\right)_{pred.}}{\left(\sigma_{60}^T\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{75}^T\right)_{exp.}-\left(\sigma_{75}^T\right)_{pred.}}{\left(\sigma_{75}^T\right)_{exp.}}\right]^2+\\ \left[\frac{\left(\sigma_{90}^T\right)_{exp.}-\left(\sigma_{90}^T\right)_{pred.}}{\left(\sigma_{90}^T\right)_{exp.}}\right]^2\end{array}\right)^{\frac{1}{2}}\times 100 \quad (27)$$

and,

$$E_{\sigma}^{C}=\frac{1}{7}\left(\begin{array}{l}\left[\frac{\left(\sigma_0^C\right)_{exp.}-\left(\sigma_0^C\right)_{pred.}}{\left(\sigma_0^C\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{15}^C\right)_{exp.}-\left(\sigma_{15}^C\right)_{pred.}}{\left(\sigma_{15}^C\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{30}^C\right)_{exp.}-\left(\sigma_{30}^C\right)_{pred.}}{\left(\sigma_{30}^C\right)_{exp.}}\right]^2+\\ \left[\frac{\left(\sigma_{45}^C\right)_{exp.}-\left(\sigma_{45}^C\right)_{pred.}}{\left(\sigma_{45}^C\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{60}^C\right)_{exp.}-\left(\sigma_{60}^C\right)_{pred.}}{\left(\sigma_{60}^C\right)_{exp.}}\right]^2+\left[\frac{\left(\sigma_{75}^C\right)_{exp.}-\left(\sigma_{75}^C\right)_{pred.}}{\left(\sigma_{75}^C\right)_{exp.}}\right]^2+\\ \left[\frac{\left(\sigma_{90}^C\right)_{exp.}-\left(\sigma_{90}^C\right)_{pred.}}{\left(\sigma_{90}^C\right)_{exp.}}\right]^2\end{array}\right)^{\frac{1}{2}}\times 100 \tag{28}$$

and,

$$E_{\sigma}^{Tb}=\frac{\left|\left(\sigma_b^T\right)_{exp.}-\left(\sigma_b^T\right)_{pred.}\right|}{\left(\sigma_b^T\right)_{exp.}}\times 100 \tag{29}$$

and,

$$E_{R}^{T}=\frac{1}{7}\left(\begin{array}{l}\left[\frac{\left(R_0^T\right)_{exp.}-\left(R_0^T\right)_{pred.}}{\left(R_0^T\right)_{exp.}}\right]^2+\left[\frac{\left(R_{15}^T\right)_{exp.}-\left(R_{15}^T\right)_{pred.}}{\left(R_{15}^T\right)_{exp.}}\right]^2+\left[\frac{\left(R_{30}^T\right)_{exp.}-\left(R_{30}^T\right)_{pred.}}{\left(R_{30}^T\right)_{exp.}}\right]^2+\\ \left[\frac{\left(R_{45}^T\right)_{exp.}-\left(R_{45}^T\right)_{pred.}}{\left(R_{45}^T\right)_{exp.}}\right]^2+\left[\frac{\left(R_{60}^T\right)_{exp.}-\left(R_{60}^T\right)_{pred.}}{\left(R_{60}^T\right)_{exp.}}\right]^2+\left[\frac{\left(R_{75}^T\right)_{exp.}-\left(R_{75}^T\right)_{pred.}}{\left(R_{75}^T\right)_{exp.}}\right]^2+\\ \left[\frac{\left(R_{90}^T\right)_{exp.}-\left(R_{90}^T\right)_{pred.}}{\left(R_{90}^T\right)_{exp.}}\right]^2\end{array}\right)^{\frac{1}{2}}\times 100\ . \tag{31}$$

and,

$$E_{R}^{Tb}=\frac{\left|\left(R_b^T\right)_{exp.}-\left(R_b^T\right)_{pred.}\right|}{\left(R_b^T\right)_{exp.}}\times 100 \tag{32}$$

The tensile and compressive yield stresses along with the tensile R-values determined from experiments required for the above error functions as they are shown in Tables (1) to (3) for Al2008-T4 and Al2090-T3.

Material	σ_0^T	σ_{15}^T	σ_{30}^T	σ_{45}^T	σ_{60}^T	σ_{75}^T	σ_{90}^T	σ_b^T
Al 2008-T4	211.67	211.33	208.5	200.03	197.3	194.3	191.56	185.0
Al 2090-T3	279.62	269.72	255	226.77	227.5	247.2	254.45	289.4

Table 1: Experimental results for Al 2008-T4 and Al 2090-T3 in tension presented by Lou et al. (2013).

Material	σ_0^C	σ_{15}^C	σ_{30}^C	σ_{45}^C	σ_{60}^C	σ_{75}^C	σ_{90}^C
Al 2008-T4	213.79	219.15	227.55	230.25	222.75	220.65	214.64
Al 2090-T3	248.02	260.75	255	237.75	245.75	263.75	266.48

Table 2: Experimental results for Al 2008-T4 and Al 2090-T3 in compression presented by Lou et al. (2013).

Material	R_0^T	R_{15}^T	R_{30}^T	R_{45}^T	R_{60}^T	R_{75}^T	R_{90}^T	R_b^T
Al 2008-T4	0.87	0.814	0.634	0.5	0.508	0.506	0.53	1.000
Al 2090-T3	0.21	0.33	0.69	1.58	1.05	0.55	0.69	0.670

Table 3: Experimental results for Al 2008-T4 and Al 2090-T3 for R-value in tension presented by Lou et al. (2013).

5 RESULTS AND DISCUSSIONS

The Modified Yld2000-2d II constructed by Modified Yld2000-2d employed as yield and Yld2000-2d used as plastic potential functions. These criteria are compared with experimental results in Figs. (1) to (6) for Al 2008-T4 (as a BCC material) and Al 2090-T3 (as a FCC material). The mechanical properties of these materials in different angles from the rolling direction are available in Tables (1) to (3). In Tables (4) and (5) parameters $\alpha_i\,(i=1-8)$, h_x and h_y along with $\beta_i\,(i=1-8)$ are computed with minimizing the error functions E_1 and E_2 in Eqs. (26) and (27) using Downhill Simplex Method for Al 2008-T4 and Al 2090-T3.

Material	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	h_x	h_y
Al 2008-T4	-0.0904	1.8021	1.8149	0.8574	-0.8184	0.0519	0.6291	1.6161	0.0173	0.0732
Al 2090-T3	1.1685	1.1393	0.5730	0.1067	0.6042	1.9914	-0.9788	1.8900	-0.0504	0.0385

Table 4: Material parameters in yield function in Modified Yld2000-2d of Al 2008-T4 and Al 2090-T3.

Material	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Al 2008-T4	0.0189	-0.0001	0.0047	0.0095	0.0079	0.0078	0.0076	0.0132
Al 2090-T3	0.1442	0.0755	0.0354	0.1284	0.1552	0.2693	0.1156	-0.0000

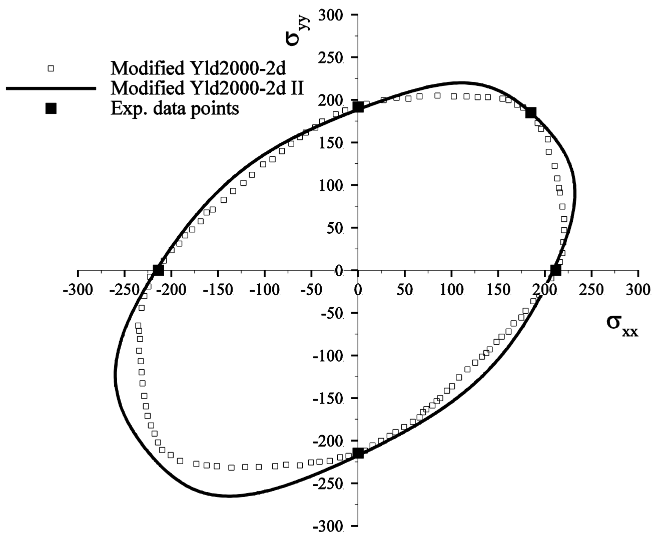
Table 5: Material parameters in plastic potential function in Yld2000-2d of Al 2008-T4 and Al 2090-T3.

Figures (1) and (2) show the $\sigma_{xx} - \sigma_{yy}$ for yield surfaces using Eq. (7) and coefficients given in Table (4) and plastic potential surfaces using Eq. (14) and coefficients given in Table (5) for Al 2008-T4 and Al 2090-T3. In these figures the differences between the yield and plastic potential surfaces for both materials can be observed because of non-AFR assumption. For both materials the yield surfaces of Modified Yld2000-2d and the Modified Yld2000-2d II are fitted to experimental data with acceptable accurately. For Al 2008-T4, both criteria are close to each other but their difference in the third quadrant is more than other parts.

Figures (3) and (4) show tensile and compressive yield stresses in different angles from the rolling direction for Al 2008-T4 and Al 2090-T3 and the obtained results are compared with experimental results. For Al 2090-T3, these criteria differ essentially and since the available experimental data are limited, it cannot be precisely said which one is more accurate.

It is observed that the Modified Yld2000-2d II is more successful than the Modified Yld2000-2d in predicting compressive yield stresses compared with experimental results. However, in predicting tensile yield stresses, the Modified Yld2000-2d is still more accurate. Figures (5) and (6) show the R-values in different angles from the rolling directions for Al 2008-T4 and Al 2090-T3. It is seen that the Modified Yld2000-2d II can predict the experimental results more precisely than Modified Yld2000-2d especially for Al 2090-T3.

(1-a)



(1-b)

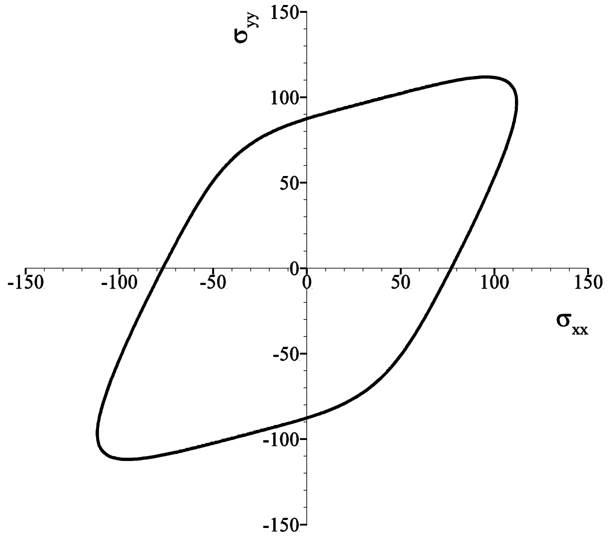
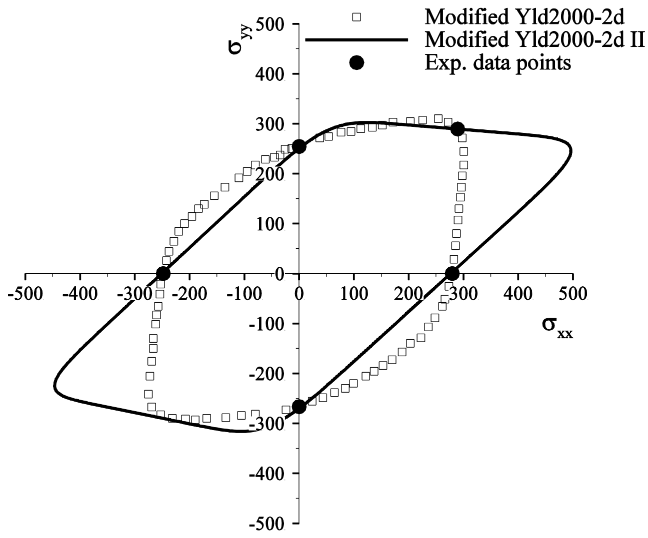


Figure 1: (a) Yield function of Modified Yld 2000-2d and Modified Yld 2000-2d II in compared with experimental results, (b) plastic potential surface of Modified Yld 2000-2d II for Al 2008-T4.

(2-a)



(2-b)

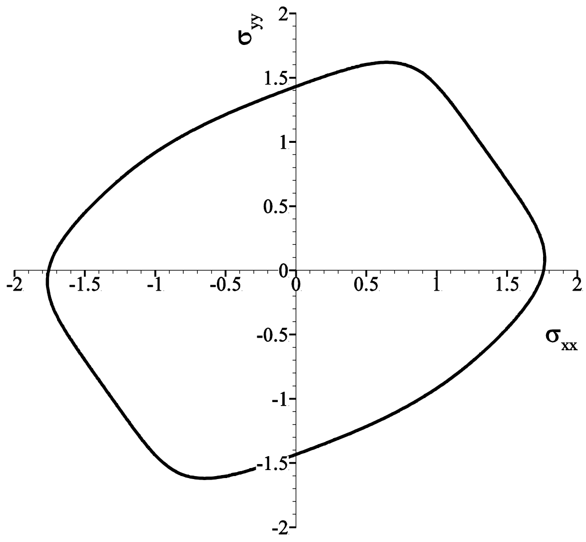
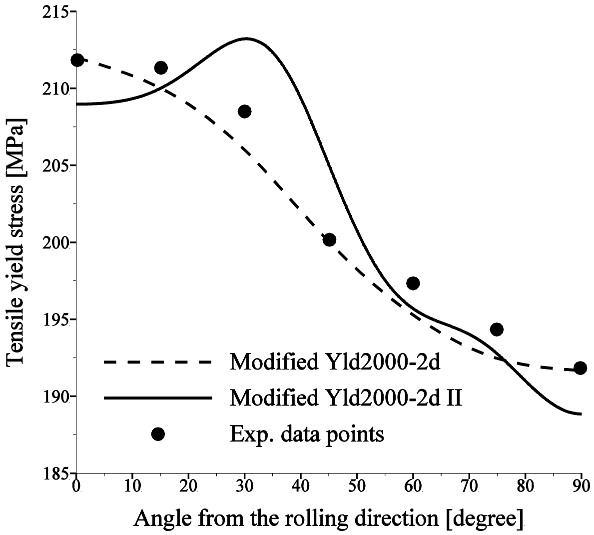


Figure 2: (a) Yield function of Modified Yld 2000-2d and Modified Yld 2000-2d II in compared with experimental results, (b) plastic potential surfaces of Modified Yld 2000-2d II for Al 2090-T3.

(3-a)



(3-b)

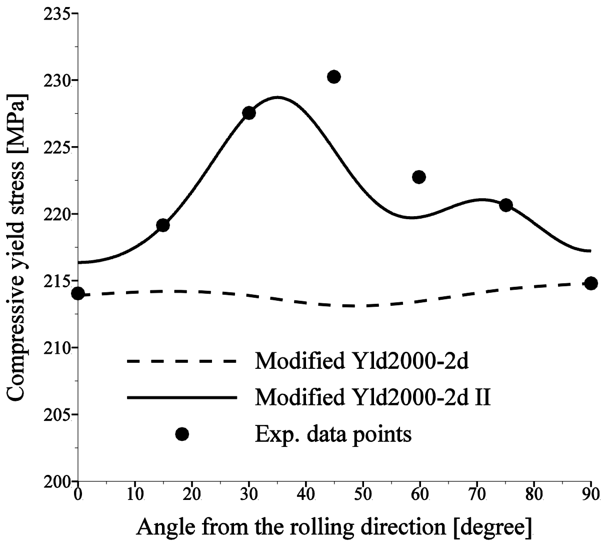
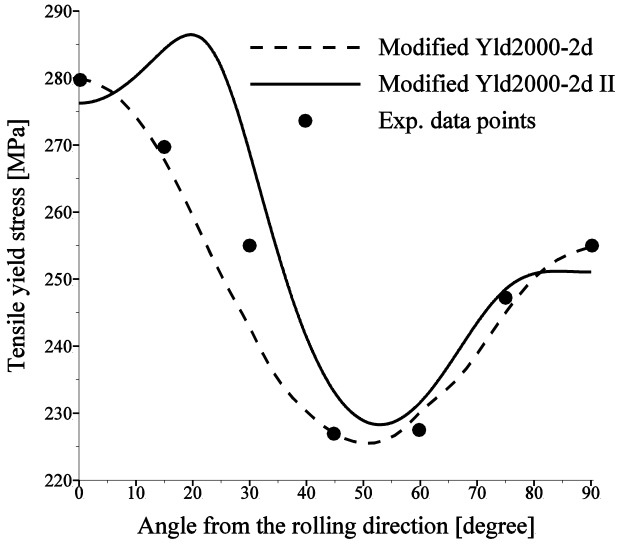


Figure 3: Comparison of the yield stress directionality for Al 2008-T4: (a) uniaxial tensile yield stress; and (b) uniaxial compressive yield stress.

(4-a)



(4-b)

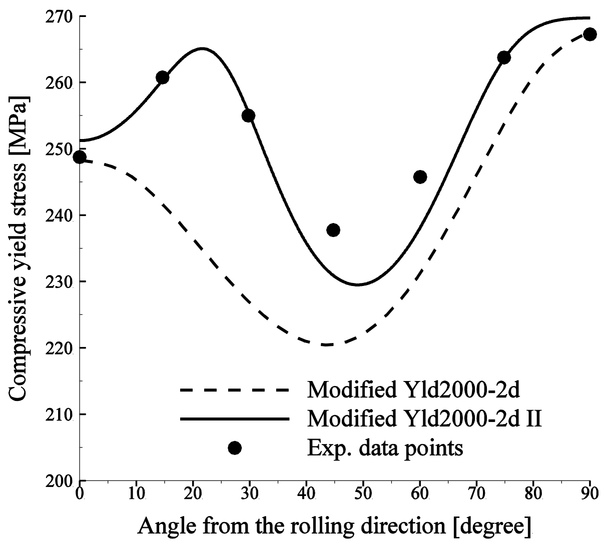


Figure 4: Comparison of the yield stress directionality for Al 2090-T3: (a) uniaxial tensile yield stress; and (b) uniaxial compressive yield stress.

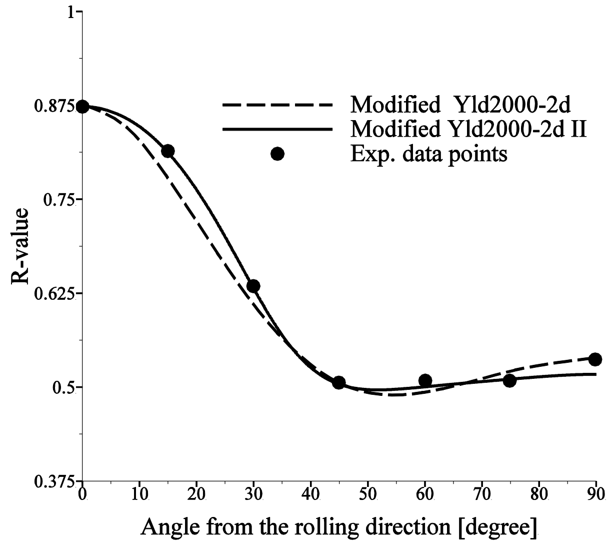


Figure 5: Comparison of the R-value directionality for Al 2008-T4.

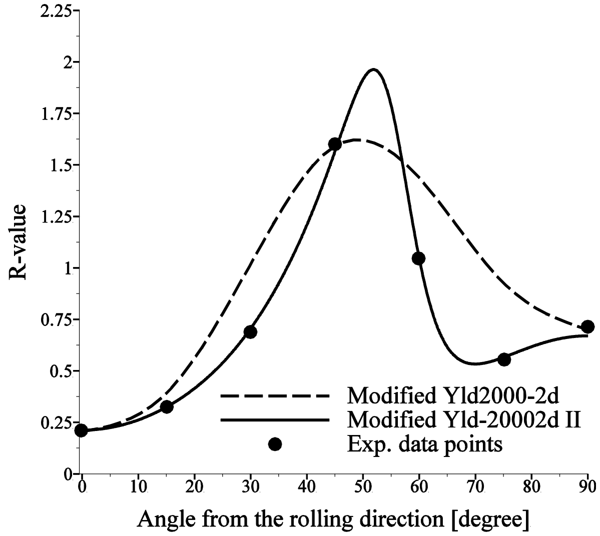


Figure 6: Comparison of the R-value directionality for Al 2090-T3.

As it is observed, to calibrate the yield function in compressive yield stresses the experimental data points are increased from two $\left(\sigma_0^C, \sigma_{90}^C\right)$ in Modified Yld2000-2d to seven in Modified Yld2000-2d II $\left(\sigma_0^C, \sigma_{15}^C, \sigma_{45}^C, \sigma_{60}^C, \sigma_{75}^C, \sigma_{90}^C\right)$ and consequently Modified Yld2000-2d II can predict the compressive yield stresses more accurately, Figs. (3-b) and (4-b). To calibrate the yield function for tensile yield stresses, however, three experimental data points $\left(\sigma_0^T, \sigma_{45}^T, \sigma_{90}^T\right)$ are selected for both criteria and it is seen that the Modified Yld2000-2d is closer to experimental data than Modified Yld2000-2d II, Figs. (3-a) and (4-a). To calibrate the yield function for biaxial yield stress $\left(\sigma_b^T\right)$ one data point is used for both criteria and they could predict the experimental data point reasonably, Figs. (1-a) and (2-a). The relative errors for these cases are computed in Tables (6) and (7).

In order to calibrate the plastic potential function in Modified Yld2000-2d II seven experimental data points are used $\left(R_0^T, R_{15}^T, R_{45}^T, R_{60}^T, R_{75}^T, R_{90}^T\right)$ which are much more than three data points in yield function of Modified Yld2000-2d $\left(R_0^T, R_{45}^T, R_{90}^T\right)$ and consequently the obtained results are more accurate than those of Modified Yld2000-2d, Figs. (5) and (6). For R_b^T , one point is selected for both criteria and they predict the experimental data point properly. The relative error for this case computed in Tables (6) and (7).

The main point of the current study is using separate yield and plastic potential functions for anisotropic pressure dependent materials, i.e. Modified Yld2000-2d (a pressure dependent criterion) for yield function and Yld2000-2d (a pressure independent criterion) for plastic potential. Using these functions increase the number of required experimental data points to calibrate the criterion from 10 for Modified Yld2000-2d and 8 for Yld2000-2d to 18 for Modified Yld2000-2d II.

In Modified Yld2000-2d and Yld2000-2d the experimental data points employed for calibration are a combination of yield stresses and R-values in different angles from rolling direction, while for the Modified Yld2000-2d II they are yield stresses and R-values for the yield and plastic potential functions, respectively. In the following the relative errors are computed in Tables (6) and (7) for Al 2008-T3 and Al2008-T4 from Eqs. (28) to (32) and compared with experimental results.

Criterion	E_σ^T	E_σ^C	E_σ^{Tb}	E_R^T	E_R^{Tb}
Modified Yld2000-2d	0.2704	1.5915	6.0536e-007	3.985	0.0187
Modified Yld2000-2d II	0.5838	0.4617	1.8179e-009	1.8179e-009	0.00076

Table 6: The obtained computation errors for Al 2008-T4 compared with experimental results (in percentage).

Criterion	E_σ^T	E_σ^C	E_σ^{Tb}	E_R^T	E_R^{Tb}
Modified Yld2000-2d	0.7350	2.4651	2.3763e-010	12.889	0.0017
Modified Yld2000-2d II	1.2207	0.6645	8.2811e-010	0.754	0.000098

Table 7: The obtained computation errors for Al 2008-T4 compared with experimental results (in percentage).

Tables (6) and (7) show that Modified Yld2000-2d II predicts σ_θ^C ,σ_b^T , R_θ^T and R_θ^T more accurately than Modified Yld2000-2d for Al 2008-T4 (a BCC material) and Al2090-T3 (a FCC material).

In the following to check the proposed criterion for strong strength differential effect, AZ31 which is a HCP material at 3% plastic strain is choosen for study. Its experimental data points is utilized from Yoon et al. (2014). Figure (7) show the yield function for AZ31 in $\sigma_{xx} - \sigma_{yy}$ and as it seen the Modified Yld2000-2d predicts experimental results with good accuracy. In Figure (8) tensile and compressive yield stresses are compared with experimental results and as it obsereved the Maodified Yld2000-2d II is succesful to predicti a the mechanical behavior of a HCP material like FCC and BCC maerials.

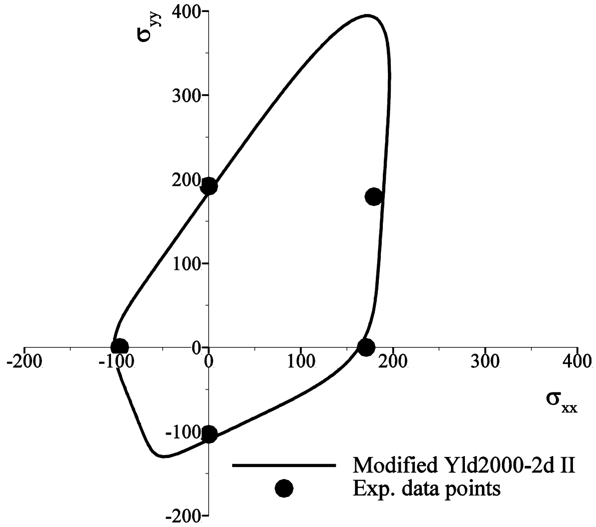
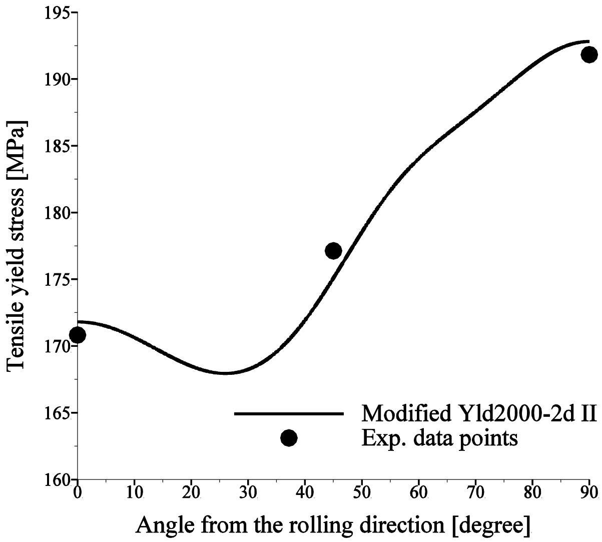


Figure 7: The Modified Yld2000-2d II and experimental data points for AZ31.

(8-a)



(8-b)

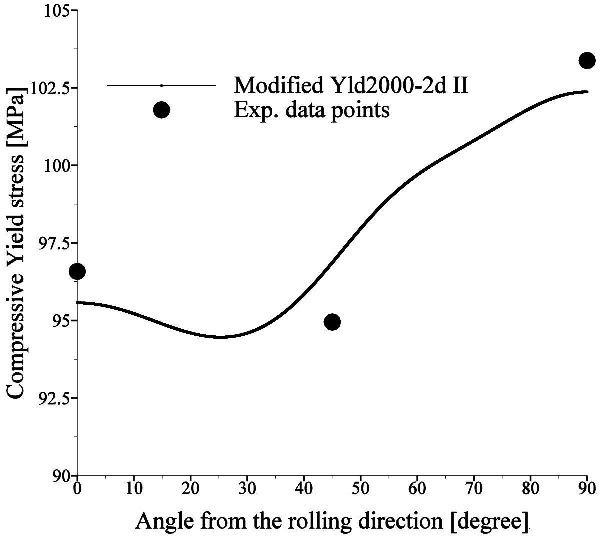


Figure 8: Comparison of the Modified Yld2000-2d II with experimental results for AZ31: (a) uniaxial tensile yield stress, (b) uniaxial compressive yield stress.

In Table 9, the obtained errors of Modified Yld2000-2d II in predicting tensile yield stress, compressive and biaxial yield stresses in compared with experimental results are shown and as it is seen the proposed criterion is also successful for a HCP material.

Criterion	E_{σ}^{T}	E_{σ}^{C}	E_{σ}^{Tb}
Modified Yld2000-2d II	0.1980	0.3896	1.9173e-010

Table 9: The obtained computation errors for AZ31 compared with experimental results (in percentage).

6 CONCLUSIONS

A non-AFR criterion with considering Modified Yld2000-2d as yield function and Yld2000-2d as plastic potential function called Modified Yld2000-2d II was presented. The yield and plastic potential functions were calibrated with ten and eight experimental data points, respectively. To verify the Modified Yld 2000-2d II in compared with experimental results, three anisoropic materials were selected contain Al 2008-T4 (a BCC material) and Al 2090-T3 (a FCC material) for slight strength differential effect and AZ31 (a HCP material) for strong strength differential effect. The obtained results showed that the Modified Yld2000-2d II (non-AFR) predicted experimental results with good accuracy and better than Modified Yld2000-2d (AFR) proposed by Lou et al. (2013) for these materials.

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