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Thermoelastic slowness surfaces in anisotropic media with thermal relaxation

Abstract

Analysis for general closed form solution of the thermoelastic waves in anisotropic heat conducting materials is obtained by using the solution technique for the biquadratic equation in the framework of the generalized theory of thermoelasticity. Obtained results are general in nature and can be applied to the materials of higher symmetry classes such as transvesely isotropic, cubic, and isotropic materials. Uncoupled and coupled thermoelasticity are the particular cases of the obtained results. Numerical computations are carried out on a fiber reinforced heat conducting composite plate modeled as a transversely isotropic media. The two dimensional slowness curves corresponding to different thermal relaxations are presented graphically and characteristics displayed are analyzed with thermal relaxations.

Keywords

Slowness Surfaces; thermoelasticity; anisotropic; coupled and thermal relaxation times.

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1 INTRODUCTION

Most materials experience volumetric variations when are subjected to temperature variations and the consequent thermal stresses developed due to temperature gradient in the surface vicinity results in micro-crack and others imperfection development at the surface of anisotropic materials. Thus owing to anisotropic material's applications in aeronautics, astronautics, plasma physics, nuclear reactors and high-energy particle and in various others engineering sciences, theory of thermoelasticity has aroused intense attention in our challenge to understand the nature of the interaction between temperature and strain fields. Main characteristics of the waves when they propagate within an anisotropic media are: phase and group velocities depend on direction – anisotropy, there is a difference between the phase velocity (propagation of the wave) and the group velocity (propagation of energy), occur shear wave splitting. Since the last century generation of waves in thermoelasticity is already known by chopping the sunlight coming onto a heat conducting metallic pate. The advent of pulsed lasers has allowed practical implementation of these ideas for the generation of energy at the surfaces of solid. The major difficulty which is inherent to thermoelastic problems comes from the fact that it is mixed elasticity problem, and for that reason complete description of the problem necessitates using the coupled wave equations of themoelasticity. Since thermal waves are highly damped at ambient temperature, and in most cases one does not necessitate to take them into account which is not true in case of thin specimens for which thermal waves are often present.

Theory of thermoelasticity when applied to stationary problems allows one to predict the nature and distribution of stress fields due to thermal fields and serve a basic tool in manufacturing engineering involving the use of temperature gradients during fabrication processes. The use of this theory in general to dynamical problems is not a trivial matter, but has been extensively described, at least for isotropic solids, by several authors Chandrasekharaiah (1986, 1998).

A large variety of wave propagation problems generated by impact execution and characterized by discontinuities of strains and thermal stresses at their front surfaces. The phase velocity or its inverse called the slowness describes the propagation of plane waves and is directed along the wave normal. Thus the notion of slowness is the reciprocal of speed (or velocity). Acoustic wave propagation in elastic media is characterized by the slowness surface. The slowness surface consists of four sheets associated with four modes of wave propagation and the two outer sheets can have zero-curvature locally. It is shown that the outmost sheet can admit extraordinary zero-curvature and the slowness curve can appear as a straight line locally. Using the perturbation method, the conditions for the extraordinary zero-curvature are derived analytically without violating the thermodynamic condition for elastic media. The results can be applied to crystals with higher symmetry and to the study of phonon focusing and surface waves. For the purposes of thermoelastic elastic wave propagation in anisotropic media, it is often practical to use the "slowness" of the wave instead the velocity.

Slowness is defined as the ratio between propagation time and propagation distance, i. e., it is simply the inverse of velocity. Slowness surface, defined by the Christoffel equation for the bulk wave propagation in anisotropic elastic media, exhibits many interesting features as in Musgrave (1970); Auld (1973). Love (1927) added another interface in order to simulate a finite thickness layer and attempted to solve the simplest case of wave interaction with it, namely that of a horizontally polarized SH wave. Studies of elastic waves in such simple and mostly isotropic systems are widely available in the books Ewing, *et al.* (1957); Achenbach 1975); Ting (1996); Graff (1975).

Theory of thermoelasticity in which the temperature field is coupled with the elastic strain fields has been widely applied in the book by Nowacki (1986). The classical theory of thermoelasticity has an unrealistic property of the diffusion type heat equation is that a swift change in temperature made at some position in the solid will be instantaneously transmitted everywhere, giving rise to an infinite propagation speed. This feature requires a modification of the Fourier law by adding a supplementary term. Lord and Shulman (1967) and Green and Lindsay (1972) extended the coupled theory of thermoelasticity by introducing the thermal relaxation time in the constitutive equations to eliminate the paradox of infinite velocity of heat propagation, thus are called generalized theories of thermoelasticity. While dealing with heat conducting anisotropic solids, the physics involved is much more intricate as three quasi-elastic waves modes are coupled to the quasi-thermal wave mode. Nevertheless, the problem is not formally intractable and has been extended and solved by various Dhaliwal and Sherief (1980) and Banerjee and Pao (1974). Many problems in generalized thermoelasticity are considered and solved by Verma (1997); Verma (2001, 2002, 2012); Bajeet(2012); Verma and Hasebe (2002);Verma and Hasebe (2004). Chiriţă (2013) studied the Rayleigh surface waves on an anisotropic homogeneous thermoelastic half space. Kumar *et al.* (2014) studied the propagation of Lamb waves in micropolar generalized thermoelastic solids with two temperatures bordered with layers or half-spaces of inviscid liquid subjected to stress-free boundary conditions in the context of generalized theory of thermoelasticity with thermal relaxation. Sura and Kanoria, (2014) studied he thermo-visco-elastic interaction due to step input of temperature on the stress free boundaries of a homogeneous visco-elastic isotropic spherical shell in the context of a new consideration of heat conduction with fractional order generalized thermoelasticity.

In general, in a heat conducting materials, four types of waves are possible in a given direction. These are associated with the directions of the particle displacement vectors and temperature. For a given wave normal in thermoelastic medium, there exists four slowness's corresponding to wave propagation along the wave normal. Considering all possible directions extending outward from a centre or origin, the set of allowable wave slowness's defines a four-sheeted slowness surface produces a centered slowness surfaces which are Centro-symmetric. These are also referred to as having different polarizations. Pure modes can be defined in different ways, but in references Wang and Li (1998) and Nayfeh (1995)define them as modes where either pure modes are defined as being modes which are either normal to (mode is longitudinal) or parallel (mode is shear) with the direction of propagation and the thermal mode. In coupled thermoelasticity modes are not pure but are skewed. Skew is defined as a measure of how far any particular mode deviates from this ideal. If the mode is pure, skew will be zero. For other modes, the skew is the angle between the polarization vector and the direction of propagation (for quasi-longitudinal modes) or the normal to the direction of propagation (for Quasi-shear modes) and for quasi-thermal modes. It is possible to compute the eigenvalues and the eigenvectors associate to such secular equation numerically, and it has been done for various classes of symmetry where the determinantal equation reduces to simper forms Banerjee and Pao (1974) and thus produce graphs for the slowness, velocity and wave surfaces and it is shown that slowness surface thermoelastic waves dependent on the frequency. Sharma (2007) studied the propagation of inhomogeneous waves, in a generalized thermoelastic anisotropic bounded medium. The slowness surfaces are identified for the waves reflected (decaying away) from the boundary. A further study on the slowness surfaces in thermoelasticity is made by Bernard Castagnede and Berthelot(1992) considering the equations of thermoelasticity. In all these study no attempt has been made to study the slowness surfaces with the thermal relaxation time and no further exclusive investigation has been made or available in the literature on heat conducting anisotropic media in the context of the equations of generalized theory of dynamic thermoelasticity, which motivated to carry out the present work to observe the behavior of slowness surfaces with thermal relaxation.

In this paper Analysis for general closed form solution of the thermoelastic waves in anisotropic heat conducting materials using the solution technique for the biquadratic equation in the framework of the generalized theory of thermoelasticity is obtained. Obtained results are general in nature and can be applied to the materials of higher symmetry classes such as transversely isotropic, cubic, and isotropic materials. Uncoupled and coupled thermoelasticity are the particular cases of the obtained results. Numerical computations are carried out on a heat conducting plate modeled as a transversely isotropic media. The two dimensional slowness curves corresponding to different thermal relaxations are presented graphically and characteristics displayed with thermal relaxations.

2 FORMULATION

Consider a set of Cartesian coordinate system $x_i = (x_1, x_2, x_3)$ and the basic field equations of generalized thermoelasticity for an infinite generally anisotropic thermoelastic medium at uniform temperature T_0 in the absence of body forces and heat sources are Verma (2002)

$$\sigma_{ij,j} = \rho \ddot{u}_{,i} \tag{1}$$

$$K_{ij} T_{,ij} - \rho C_e (\dot{T} + \tau_0 \ddot{T}) = T_0 \beta_{ij} [\dot{u}_{i,j} + \tau_0 \ddot{u}_{i,j}] \tag{2}$$

$$\beta_{ij} = C_{ijkl} \alpha_{kl} \tag{3}$$

K_{ij} and β_{ij} are the thermal conductivities and thermoelastic coupling tensors respectively, α_{kl}, ρ, τ_0 and C_e are the linear thermal expansion, density, thermal relaxation time and specific heat at constant strain of the material. Comma notation is used for spatial derivatives and superposed dot represents differentiation with respect to time.
Strain-displacement relation

$$e_{ij} = (u_{i,j} + u_{j,i})/2. \tag{4}$$

3 ANALYSIS

Assume that solutions to the equations (1) and (2) are expressed by

$$(u_j, T) = (U_j, \Theta) \exp[i\xi(v_1 x_1 + v_2 x_2 + v_3 x_3 - vt), \ j = 1, 2, 3. \tag{5}$$

where ξ is the wave number, v is the phase velocity ($= \omega/\xi$), ω is the circular frequency, U_j and Θ are the constants related to the amplitudes of displacement and temperature, v_k, k = 1, 2, 3 are the components of the unit vector giving the direction of propagation.

Substituting equation (5) into equations (1) and (2), we have

$$(\Gamma_{ik} - \rho v^2 \delta_{ik})U_k + i\omega^{-1} v \beta_{ij} v_j \Theta = 0, \tag{6}$$

$$i\omega v T \beta_{ij} \tau v_j U_i - (K_{ij} v_i v_j - \rho_n C_e \tau v^2)\Theta = 0, \tag{7}$$

where $\tau = i\omega^{-1} + \tau_0$, δ_{ik} is the Kronecker delta, and Γ_{ik} are the Christofied stiffness as follows:

$$\Gamma_{ik} = \Gamma_{ki} = C_{ijkl} v_j v_l. \tag{8}$$

The equations (6) and (7) provide a non-trivial solution for U_j and Θ if the determinant of their coefficients vanishes. This leads to

$$\rho v^2 \det[\bar{\Gamma}_{ik} - \rho v^2 \delta_{ik}] + i\omega\{K C_e^{-1} \det[\Gamma_{ik} - \rho v^2 \delta_{ik}] - \tau_0 \rho v^2 \det[\bar{\Gamma}_{ik} - \rho v^2 \delta_{ik}]\} = 0, \tag{9}$$

where $K = K_{ij} v_j v_l$,

$$\bar{\Gamma}_{ik} = \Gamma_{ik} + T_0 \beta_{ip} \beta_{kq} v_p v_q / (\rho C_e) \tag{10}$$

are the isothermal acoustical tensor, the effective thermal conductivity for linear heat flow in the direction of (v_1, v_2, v_3) and the isentropic acoustical tensor respectively. Clearly (9) represents an eigenvalue problem, where the phase velocities $\mathbf{v}$ are the eigenvalues, and the U_j vectors (polarization vectors) are the eigenvectors. In general, there will be four phase velocities, accompanied by three polarization vectors and thermal variation. These phase velocities and polarizations define a single (quasi)longitudinal and two (quasi)shear and a thermal modes. Explicitly, the eigenvalue problem is as follow

$$\det[A_{ik} - \rho v^2 I] = \begin{vmatrix} A_{11} - \rho_n v^2 & A_{12} & A_{13} \\ A_{21} & A_{22} - \rho v^2 & A_{23} \\ A_{31} & A_{32} & A_{33} - \rho v^2 \end{vmatrix} \tag{11}$$

where $A_{ik} = \Gamma_{ik}$ or $\bar{\Gamma}_{ik}$.

Specializing, the above equations for heat conducting orthorhombic materials, the characteristic equation of the eigenvalue problem defined in above equation becomes for orthorhombic symmetry, the determinant

$$\det(M_{ij}) = 0, \quad i, j = 1, 2, 3, 4. \tag{12}$$

where

$$M_{11} = c_{11} v_1^2 + c_{66} v_2^2 + c_{55} v_3^2 - \rho v^2, M_{12} = (c_{12} + c_{66}) v_1 v_2$$
$$M_{13} = (c_{13} + c_{55}) v_3 v_1, M_{14} = v_1$$
$$M_{22} = c_{66} v_1^2 + c_{22} v_2^2 + c_{44} v_3^2 - \rho v^2, M_{23} = (c_{23} + c_{44}) v_2 v_3$$
$$M_{33} = c_{55} v_1^2 + c_{44} v_2^2 + c_{33} v_3^2 - \rho v^2, M_{24} = \bar{\beta}_2 v_2, M_{34} = \bar{\beta}_3 v_3$$
$$M_{41} = \varepsilon \omega^* \rho v^2 \beta_1 v_1 \tau, M_{42} = \varepsilon \omega^* \rho v^2 v_2 \bar{\beta}_2 \tau,$$
$$M_{43} = \varepsilon \omega^* \rho v^2 v_3 \bar{\beta}_3 \tau, M_{44} = K_1 v_1^2 + K_2 v_1^2 + K_3 v_3^2 - \omega^* \tau \rho v^2$$

Relationship describing the wave surfaces in each plane can be derived from (12). Here we are considering X_2-X_3 plane:

$$v_t = \sqrt{(c_{44}\sin^2\varphi + c_{66}\cos^2\varphi)/\rho} \tag{13}$$

and v_{ql} , v_{qt} and v_{qT} are roots of

$$C_e\tau v^6 + A_1 v^4 + A_2 v^2 + A_3 = 0 \tag{14}$$

$$A_1 = \big((c_{11}+c_{55})\cos^2\varphi + (c_{33}+c_{55})\sin^2\varphi\big)C_e\tau + (\cos^2\varphi + \overline{\beta}_3^2\sin^2\varphi)G_1 + K_1\cos^2\varphi + K_3\sin^2\varphi$$

$$\begin{aligned} A_2 = {} & (-c_{11}c_{55}\cos^4\varphi + (2c_{13}c_{55} + c_{13}{}^2 - c_{11}c_{33})\cos^2\varphi\sin^2\varphi - c_{33}c_{55}\sin^4\varphi)F \\ & + \big[-c_{55}\cos^4\varphi + \big(-c_{11}\overline{\beta}_3^2 + 2(c_{13}+c_{55})\overline{\beta}_3 - c_{33}\big)\cos^2\varphi\sin^2\varphi \\ & - c_{55}\overline{\beta}_3^2\sin^4\varphi\big]G_1 - \big(K_1\cos^2\varphi + K_3\sin^2\varphi\big)\big((c_{11}+c_{55})\cos^2\varphi + (c_{33}+c_{55})\sin^2\varphi\big) \end{aligned}$$

$$A_3 = \big(c_{11}c_{55}\cos^4\varphi - (2c_{13}c_{55} + c_{13}{}^2 - c_{11}c_{33})\cos^2\varphi\sin^2\varphi + c_{33}c_{55}\sin^4\varphi\big)\big(K_1\cos^2\varphi + K_3\sin^2\varphi\big)$$

The angle φ is measured from X_i to X_j such that $i<j$.

$$G = \varepsilon\omega^*\rho\tau\,,\quad F = c_{11}\tau\,,\quad \omega^* = \frac{c_{11}C_e}{k_1}\,,\quad \varepsilon = \frac{\beta_1^2 T_0}{\rho c_{11}C_e}\,,\quad \overline{\beta}_2 = \beta_2/\beta_1\,,\ \overline{\beta}_3 = \beta_3/\beta_1\,, \tag{15}$$
$$\tau = \tau_0 + i/\omega$$

Similar relationship describing the wave surfaces in X_1-X_2 and X_1-X_3 planes can be obtained and discussed.

4 UNCOUPLED AND COUPLED THERMOELASTICITY

If the thermal relaxation time is taken zero i.e. $\tau_0 = 0$, then the analysis in section 2 reduces to the definition of classical coupled thermoelasticity. Similarly, if the coupling constant is taken zero i.e., $\varepsilon = 0$, then the analysis reduces to the definition of uncoupled generalized thermo elasticity.

5 NUMERICAL COMPUTATION RESULTS AND DISCUSSION

The numerical computation is carried out over a cobalt plate.
Physical data for this material are given as:

T_0	$298\ ^0\mathrm{K}$	β_1	$7.040\times10^6\,Nm^{-2}\deg^{-1}$
ρ	$8.836\times10^3\,kg\ m^{-3}$	β_2	$6.900\times10^6\,Nm^{-2}\deg^{-1}$
C_{11}	$3.071\times10^{11}\,Nm^{-2}$	C_e	$4.270\times10^2\,Jkg^{-1}\deg^{-1}$
C_{12}	$1.650\times10^{11}\,Nm^{-2}$	K_1	$0.6900\times10^2\,Wm^{-1}\deg^{-1}$
C_{13}	$1.027\times10^{11}\,Nm^{-2}$	K_3	$0.6900\times10^2\,Wm^{-1}\deg^{-1}$
C_{33}	$3.581\times10^{11}\,Nm^{-2}$	ε	0.129
C_{44}	$0.7550\times10^{11}\,Nm^{-2}$	ω^*	$1.880\times10^{12}\,s^{-1}$

Table 1: Physical data for a single crystal of cobalt.

Figures 1-3 depict the slowness surfaces for the thermoelastic single crystal of cobalt whose physical data is given in Table-1. Each figure exhibit the four surfaces one for quasi-longitudinal, two for quasi-shear and one for quasi-thermal. Figure 1-3 represent the slowness surfaces for when thermal relaxation time increases from $1\times10^{-13}\sec s$ to $1\times10^{-11}\sec s$. From the figures it is observed that on increasing the thermal relaxation time slowness surfaces for quasi-shear modes have no effect on varying the thermal relaxation time, whereas quasi-longitudinal and quasi-thermal modes are highly affected by the thermal relaxation variations. The inner two curves correspond to the quasi-longitudinal and quasi-thermal wave modes whereas the outer two curves correspond to the two quasi-shear wave modes. Shapes of all the four slowness curves are circular or elliptical.

The innermost curve corresponds to the quasi-thermal (qT) wave mode, next from the inner is the quasi-longitudinal(qL) mode the one with dimples to the quasi-shear (qSV) wave, and the outer elliptical surface to the quasi-shear (qSH) wave. In slowness space, the four slowness surfaces for the qT, ql, qSV, and qSH waves are from innermost to outermost are obtained.

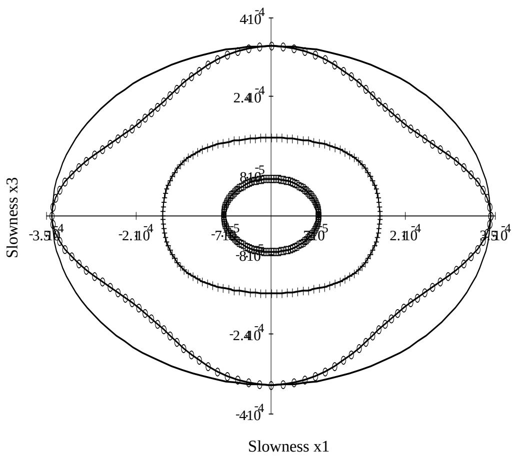


Figure 1: Thermoelastic slowness surfaces for crystal of cobalt when thermal relaxation time is $1\times10^{-13}\sec s$.

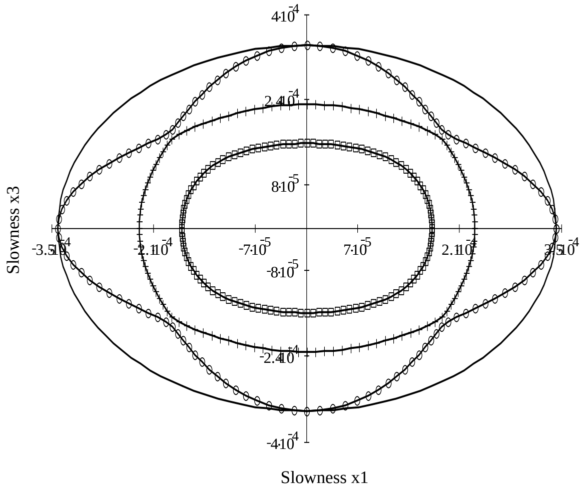


Figure 2: Thermoelastic Slowness surfaces for crystal of cobalt when thermal relaxation time is $1\times10^{-12}\,\sec s$.

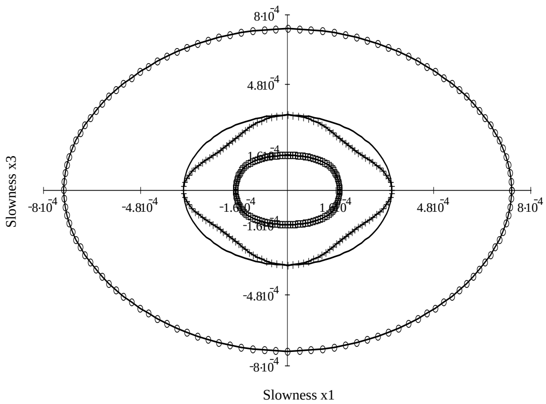


Figure 3: Thermoelastic Slowness surfaces for crystal of cobalt when thermal relaxation time is $1\times10^{-11}\,\sec s$.

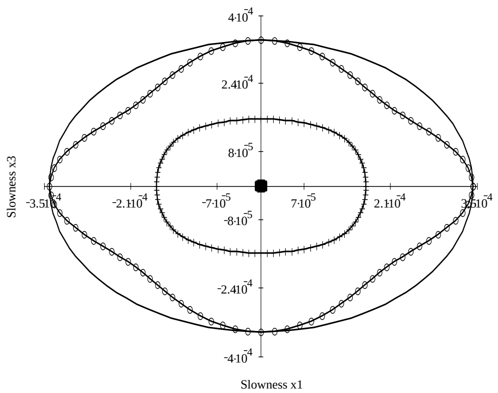


Figure 4: Thermoelastic Slowness surfaces for crystal of cobalt when thermal relaxation time is equalto zero.

Results for possessing transverse isotropy, whose x_1 axis is normal to the plane of isotropy, can be easily obtained by noting the additional conditions imposed by symmetry, namely

$$c_{33} = c_{22},\ c_{13} = c_{12},\ c_{55} = c_{66},\ 2c_{44} = c_{22} - c_{23}$$
$$\alpha_{33} = \alpha_{22}, \alpha_{13} = \alpha_{12},\ \alpha_{55} = \alpha_{66}, 2\alpha_{44} = \alpha_{22} - \alpha_{23}$$

And for cubic symmetry

$$c_{11} = c_{22} = c_{33},\ \ c_{12} = c_{13} = c_{23},\ c_{44} = c_{55} = c_{66} \text{ and } \alpha_{11} = \alpha_{22} = \alpha_{33}$$

Finally, for the isotropic case

$$c_{11} = c_{22} = c_{33} = \lambda + 2\mu,\ c_{12} = c_{13} = c_{23} = \lambda$$
$$c_{44} = c_{55} = c_{66} = \mu, \beta_j = \beta, K_j = K,\ (j = 1,2,3.)$$

For thermoelastic isotropic solids, the wave-velocity surfaces are concentric spheres in the same way as in corresponding elastic media, where transverse (t) and quasi-transverse (qt) surfaces coincide and all waves are pure mode. In generalized thermoelasticity all waves are not in pure mode, longitudinal and a thermal wave-velocity surface are coupled, and exists as quasi- longitudinal (ql) and quasi-thermal (qth) mode. Shear wave mode decoupled and is not affected by the thermal fields. Figures 5-8 exhibit polar diagrams of phase velocity (m/s) for an isotropic aluminum material with different thermal relaxation times. Whereas Figure 9 shows polar diagrams of phase velocity (m/s) for an isotropic aluminum material when coupling constant is zero

In Figure 5, wave-velocity surface corresponding to quasi-longitudinal mode (ql) is a sphere with greater radius than quasi-thermal (qth)mode wave-velocity surface are drawn, when thermal relaxation time $\tau_0 > 1.363\times10^{-13}$,, and they are in the order $ql > t > qth$, this shows that longitudinal wave velocity will exceed thermal wave velocity, and the wave velocity of transverse wave velocity lies between longitudinal and thermal wave velocities. When $\tau_0 = 1.379\times10^{-13}$, $ql > t(= qth)$, longitudinal wave velocity exceed thermal wave velocity, and transverse wave velocity become equal to thermal wave velocities is shown in Figure 6.

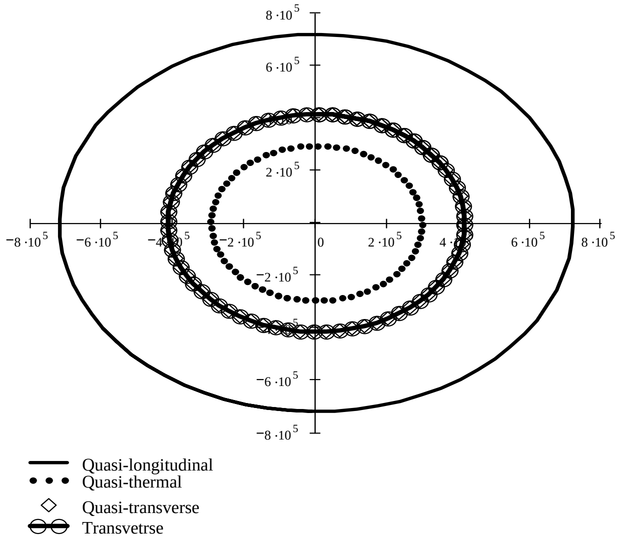


Figure 5: Polar diagram of phase velocity (m/s) for an isotropic aluminum material when coupling constant $\tau_0 = 1.363\times10^{-13}s$.

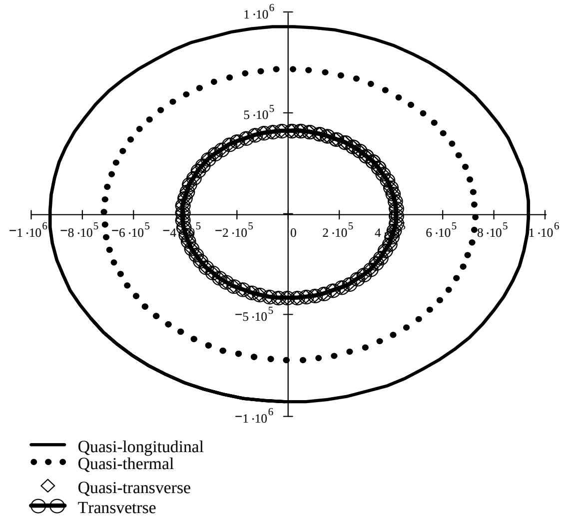


Figure 6: Polar diagram of phase velocity (m/s) for an isotropic aluminum material when coupling constant $\tau_0 = 1.379\times10^{-13}$.

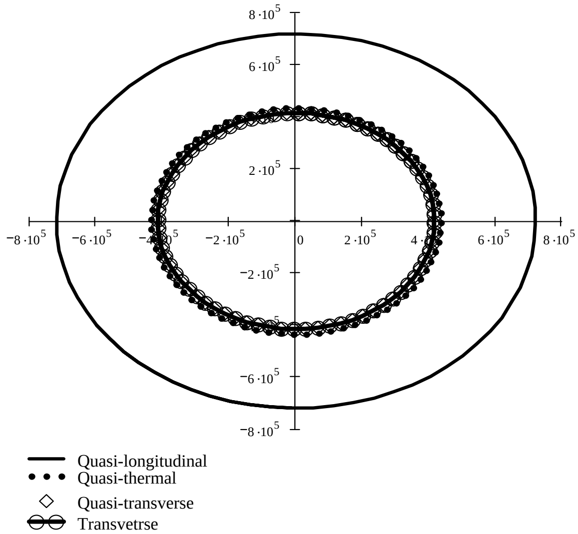


Figure 7: Polar diagram of phase velocity (m/s) for an isotropic aluminum material when coupling constant $\tau_0 = 1.0\times10^{-14}s$.

When $\tau_0 = 2.293\times10^{-14}$, $ql = qth\ (> t)$, and at $\tau_0 = 2.21\times10^{-14}$, quasi-longitudinal and quasi-thermal mode conversion take place is shown in Figure 7.

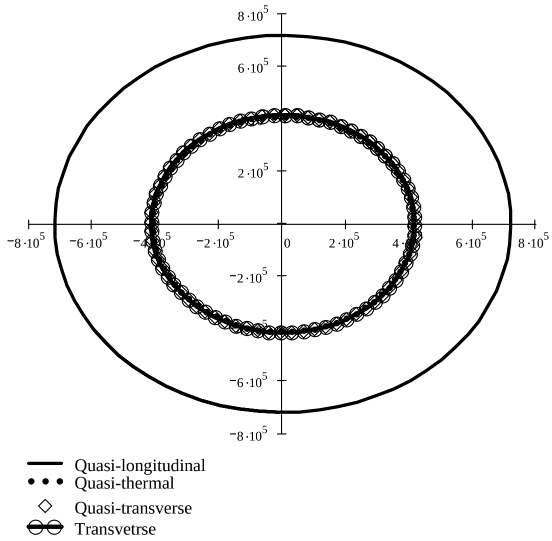
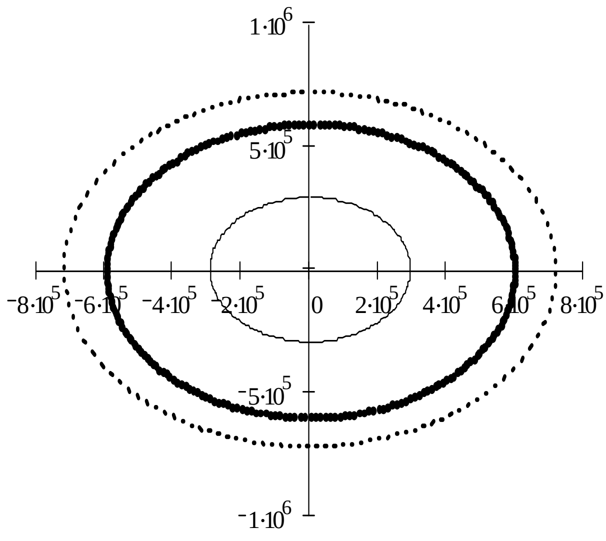


Figure 8: Polar diagram of phase velocity (m/s) for an isotropic aluminum material when coupling constant $\tau_0 = 2.293\times10^{-14}s$.



.... Wave Surface (ql)
*** Wave Surface (qth)
– Wave surface (transverse t)

Figure 9: Polar diagram of phase velocity (m/s) for an isotropic aluminum material when coupling constant is zero.

From the above discussion it is observed that thermo-mechanical stability requires that vq_{thermal}, $2.938.10^5(m/s) < vq_{\text{thermal}} < 7.208.10^5(m/s)$, while thermal relaxation time $2.293\times10^{-14}\,\text{secs} < \tau_0 < 1.379\times10^{-13}\,\text{secs}$, Thus $q_{l,}\ q_{th}$ and t surfaces cannot cross in the thermo isotropic–solid case, for mechanical stability requires that v_l exceed $v_t\sqrt{4/3}$. Thus l and t surfaces cannot cross in the isotropic–solid case. Quasi-longitudinal and quasi-thermal surfaces mean that a transverse wave velocity will exceed a longitudinal wave velocity. A longitudinal- transverse mode conversion means that both longitudinal and transverse modes exit on the wave surfaces.

6 CONCLUSIONS

In this article analysis for general closed form solution of the thermoelastic waves in anisotropic heat conducting materials is obtained and the solution technique for the secular equation in the framework of the generalized theory of thermoelasticity is employed. Obtained results are general in nature and can be applied to the materials of higher symmetry classes such as transversely isotropic, cubic, and isotropic materials. Uncoupled and coupled thermoelasticity are the particular cases of the obtained results. Numerical computations are carried out for a crystal of cobalt modeled as a transversely isotropic media. The two dimensional slowness curves corresponding to different thermal relaxations are presented graphically and characteristics displayed are analyzed with thermal relaxations. Slowness surfaces for the thermoelastic single crystal of cobalt are obtained at different values of thermal relaxation time. Each figure exhibit the four surfaces one for quasi-longitudinal, two for quasi-shear and one for quasi-thermal. It is also observed that on increasing the thermal relaxation time, there is no effect of thermal relaxation time on slowness surfaces for quasi-shear modes, whereas quasi-longitudinal and quasi-thermal modes are highly affected by the thermal relaxation variations. Further inner two curves correspond to the quasi-longitudinal and quasi-thermal wave modes whereas the outer two curves correspond to the two quasi-shear wave modes and the shapes of all the four slowness curves are circular or elliptical.

For thermo elastic isotropic solids, the wave-velocity surfaces are concentric spheres in the same way as in its counterpart elastic media, where transverse (t) and quasi-transverse (qt) surfaces coincide and all waves are pure mode. In generalized thermoelasticity all waves are not in pure mode, longitudinal and a thermal wave-velocity surface are coupled, and exists as quasi-longitudinal (ql) and quasi-thermal (qth) mode. Shear wave mode decoupled and is not affected by the thermal fields. It is observed that thermo-mechanical stability in the case isotropic material requires that quasi-longitudinal velocity mode should exceed $\sqrt{4/3}$ times the quasi-transverse velocity. It is observed that for thermo-mechanical stability in case of quasi-thermal requires, $2.938.10^5(m/s) < \text{quasi-thermal} < 7.208.10^5(m/s)$, and thermal relaxation time should be in the range $2.293\times10^{-14}\,\text{secs} < \tau_0 < 1.379\times10^{-13}\,\text{secs}$. Further studies using the above equations and results can be made by varying other thermal fields.

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